UNIT-I
INTRODUCTION

The word "soil" is derived from the latin word solium which according to webster's dictionary means the upper layer of the earth that may be dug or plowed : specifically the loose surface material of the earth in which plants grow.

The term 'soil' in the soil engineering is defined as an unconsolidated material composed of solid particles produced by the disintegration of rocks. The void space between the particles may contain air, water or both. The solid particles may contain organic matter. The soil particles can be separated by such machanical means as agitation in water.

Soil Engineering & Geotechnical Engineering :-

Soil engineering in an applied science dealing with the applications of principles of soil mechanics to partical problems. It has a much wider scope than soil mechanics, as it deals with all engineering problems realted with soils. It includes site investigations, design and construction of foundations, earth - retaining structures and earth structures.

Geotechnical engineering is a broader term which includes soil engineering, rock mechanics and geology. This term is used synonymously with soil engineering in this text. The term "soil mechanics" was coined by Dr. Karl Terzaghi in 1925

FORMATION OF SOILS :-

Soils are formed by either

A) physical disintegration

B) chemical decomposition of rocks.

A) Physical disintergration :-

Physical disintergration or mechanical weathering of rocks occurs due to the following physical processes :

1. Temperature changes :- Different minerals of rocks have different co-efficients of thermal expansion. Unequal expansion and contraction of these minerals occur due to temperature changes. When the stresses induced due to such changes are repeated many

times, the particles get detached from the rock and the soils are formed.

2. Wedging action of Ice :- water in the pores and minute cracks of rocks gets frozen in very cold climates. As the volume of Ice formed in more than that of water, expansion occurs. Rocks get broken into pieces when large stress develop in the cracks due to weding action of the ice formed.

3. spreading of roots of plants :- As the roots trees and shrubs grow in the cracks and fissures of the rocks, forces act on the rocks. The segments of the rocks are forced apart and disintegration of rocks occurs.

4. Abrasion :- As water, wind and glaciers move over the surface of rocks, abrasion and scouring takes place. It results in the formation of soil.

In all the process of physical disintegration, there is no change in the chemical composition. The soil formed has the properties of the parent rocks. coarse grained soils, such as gravel and sand, are formed by the process of physical disintegration.

B) Chemical Decomposition :- When chemical decomposition or chemical weathering of rocks takes places original rocks minerals are transformed into new minerals by chemical reactions. The soils formed do not have the properties of the parent rocks. The following chemical processes generally occur in nature.

1. Hydration :- In hydration, water combines with the rocks minerals and results in the formation of a new chemical compound. The chemical reaction causes a change in volume and decomposition of rocks into small particles.

2. Carbonation :- It is a type of chemical decomposition in which carbon dioxide in the atmosphere combines with water to form carbonic acid. The carbonic acid reacts chemically with rocks and causes their decomposition.

3. Oxidation :- Oxidation occurs when oxygen ions combine with minerals in rocks. Oxidation results in decomposition of rocks. Oxidation of rocks is some what similar to rusting of steel.

4. Solution:- Some of the rock minerals from a solution with water when they get dissolved in water. Chemical reaction takes place in the solution and the soils are formed.

5. Hydrolysis:- It is a chemical process in which water gets dissociated into H^+ and OH^- ions. The hydrogen cations replace the matalic ions such as calcium, sodium and potassium in the rock minerals and soils are formed with anew chemical decomposition.

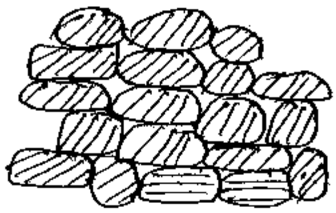
chemical decomposition of rocks results in formation of clay minerals. These clay minerals imparts plastic properties to soils. clayey soils are formed by chemical decomposition.

SOIL STRUCTURES:-

The geometrical arrangement of soil particles with respect to one another is known as soil structure. The soil in nature have different structures depending upon the particle size and the mode of formation. The following types of structures are usually found. The first two types are for coarse grained soils and types ③ and ④ for clays. Types ⑤ and ⑥ are for mixed soils.

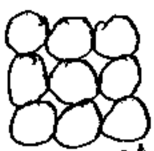
1. Single - grained structure :-

cohesionless soils, such as gravel and sand are composed of bulky grains in which the gravitational forces are more predominant that surface forces. When decomposition of these soils occurs, that particles settle under gravitational forces and take an equilibrium position as shown in the below figure. Each particle is in contact with those sourrounding it. The soil structure so formed is known as single grained structure.

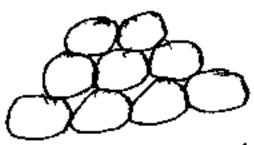


1. Single grained structure

Depending upon the relative position of the particle the soil may have a loose structure or a derice structure. Loosest condition, the void ratio is 0.90 and the densert condition the void ratio is 0.35



a) Loose structure

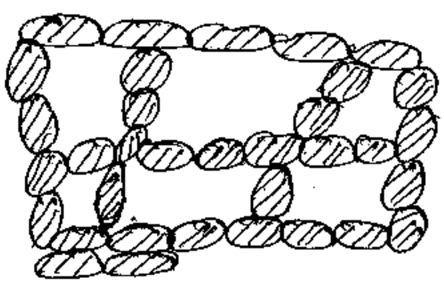


b) Dense structure.

2. Honey - comb structure :-

It is possible for fine sands or silts to get deposited such that the particles when setting develop a particle - to - particle contact that bridges over large voids in the soil mass. The particles wedge between one another into a stable condition and form a skelton like an arch to carry the weight of overlying material. The structure so formed is known as honey comb structure.

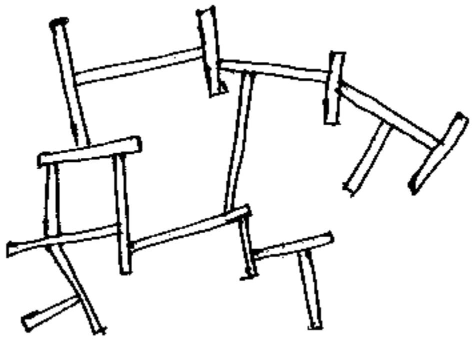
The honey - comb structure usually develops when the particles size is between 0.002 mm and 0.02 mm



Honey - comb structure.

3. Flocculated structure :-

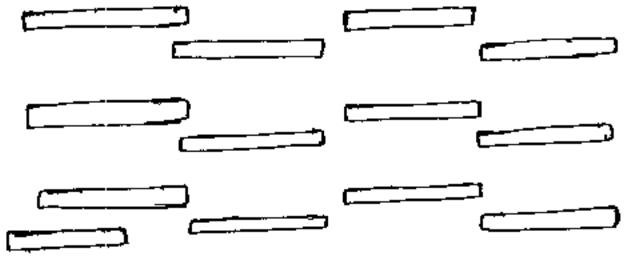
Flocculated structure occurs in clays. The clay particles have large surface and therfore, the electrical forces are important in such soils. The clay particles have a negative charge on the surface and a positive charge on the edges. Interparticle contact develops between the positevly changed edges and the negatively charged faces. This results in a flocculated structure.



Flocculated structure.

4. Dispersed structure :-

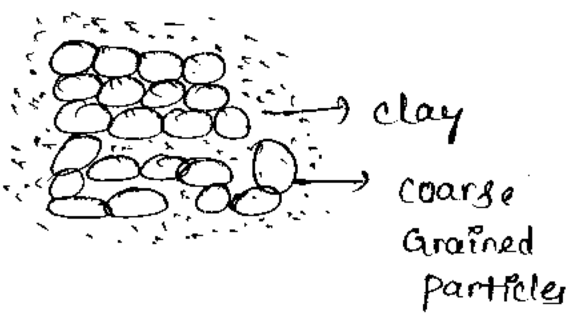
Dispersed structure develops in clays that have been reworked or remoulded. The particles develop more or less a parallel orientation. clay deposits with a flocculent structure when transported to other places by nature or man get remoulded. Remoulding converts the edge-to-face orientation to face-to-face orientation. The dispersed structure is formed in nature when there is a net repulsive fore between particles.



Dispersed structure.

5. Coarse - grained skeleton :-

A coarse grained skeleton is a composite structure which is formed when the soil contains particles of different types. when the amount of bulky, cohesionless particles is large compared with that of fine - grained clayey particles, the bulky grains are in particle - to - particle contact. These particles form a formwork or skelton. The space between the bulky grains is occupied by clayey particles known as binders



a) Coarse grained skeleton

b) clay matrix.

6. clay matrix :- clay matrix is also a composite structure formed by soil of different types However, in this case, the amount of clay particles is very large as compared with bulky coarse-grained particles. The clay forms a matrix in which bulky grains appear floating without touching one another.

The soil with a clay matrix structure have almost the same properties as clay. Their behaviour is similar to that of an ordinary clay deposite. However, they are more stable as disturbance has very little effect on the soil formation with a clay-matrix structure.

Weight - volume relationships :-

1. Bulk unit weight (γ) :- It is defined as the ratio of weight per unit volume of the soil mass. It is defined by 'γ' (Gama)

$$\boxed{\therefore \gamma = \frac{W}{V}}$$

Units :- N/m^3 (or) kN/m^3

2. Unit weight of solids (γ_s):-

It is defined as the ratio of weight of soil solids to that of the volume of soil solids

$$\boxed{\gamma_s = \frac{W_s}{V_s}}$$

It is also called as absolute unit weight of soil.

3. Unit weight of water (γ_w):-

It is the ratio of the weight of water to the volume of water.

$$\boxed{\gamma_w = \frac{W_w}{V_w}}$$

$w = 1000\ kg/m^3$ (or) $9.81\ kN/m^3$

4. Dry unit weight (γ_d):-

It defined as the ratio of weight of soil solids per unit total volume.

$$\boxed{\gamma_d = \frac{W_s}{V}}$$

5. Saturated unit weight:-

$$\boxed{\gamma_{sat} = \frac{W_{sat}}{V}}$$

The saturated unit weight is the bulk unit weight when the soil is saturated.

6. <u>Submarged unit weight:-</u>

It is defined as the ratio of the submarged weight of soil solids to that of the unit total volume of the soil.

$$\boxed{\gamma' = \frac{(W_s)_{sub}}{V}}$$

$(W_s)_{sub}$ = weight of solid particles in air weight of water displaced by the solids

$$= W_s - V_s \gamma_w$$

$$= W - W_w - V_s \gamma_w$$

$$= W - \gamma_w V_w - \gamma_w V_s$$

$$= W - \gamma_w (V_w + V_s)$$

$$(W_s)_{sub} = W - V\gamma_w$$

Dividing throughout by V

$$\frac{(W_s)_{sub}}{V} = \frac{W}{V} - \frac{V\gamma_w}{V}$$

$$\boxed{\gamma' = \gamma_{sat} - \gamma_w}$$

$$\gamma_s > \gamma_{sat} > \gamma > \gamma_d > \gamma'$$

ADSORBED WATER :-

The water held by electro-chemical forces existing on the soil surface is adsorbed water. As the adsorbed water is under the influence of electrical forces, its properties are different from that of normal water. It is much more viscous, and its surface tension is also greater. It is heavier than normal water. The boiling point is higher, but the freezing point is lower than that of the normal water.

The thickness of the adsorbed water layer is about 10 to 15 A° for colloids but may be upto. 200 A° for silts. The attractive forces between the adsorbed water and the soil surface decrease exponentially with the distance untill the double layer merges into normal water. The adsorbed water exists in an almost solidified state. The pressure required to pull away the adsorbed water layer from the soil surface is very high it may be as high as 10,000 atmospheres.

Adsorbed water imparts plasticity characteristics to soils. The adsorbed water depends upon the clay minerals present in the soil. The presence of highly active clay minerals in necessary to give the soil plasticity. The fine-grained soil with out clay minerals may develop cohesion if the particle size is very small, but these soils cannot be moulded into small threads as these are not plastic.

RELATIVE DENSITY:-

The most important index aggregate property of a cohesionless soil is its relative density. The engineering properties of a mass of cohesionless soil depend to a large extent on its relative density (D_r) also known as density index (I_D). The relative density is defined as

$$D_r = \frac{e_{max} - e}{e_{max} - e_{min}} \times 100$$

where e_{max} = Maximum void ratio of the soil in the loosest condition

e_{min} = Minimum void ratio of the soil in the densest condition

e = void ratio in the nutral state.

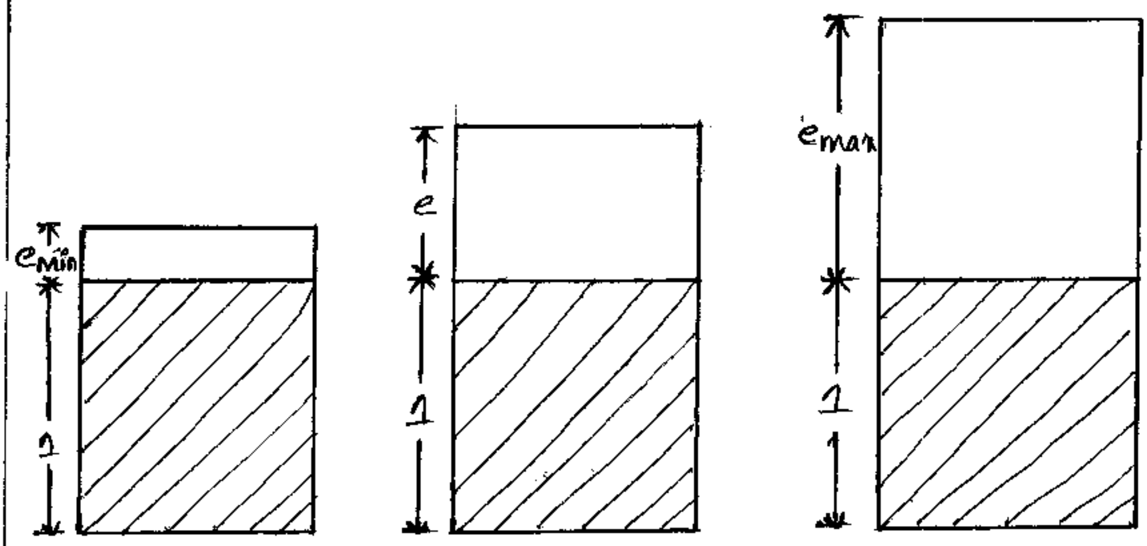
The relative density of a soil gives a more clear idea of the denseness than does the void ratio. Two types of sand having the same void ratio may have entirely different state of denseness and engineering properties. However, if the two sands have the same relative density, they usually behave in identical manner.

The relative density of a soil indicates how it would behave under loads. If the deposits is dense. It can take heavy loads with very little settlements. Depending upon the relative density, the soils are generally divided into 5 categories.

Denseness	very Loose	Loose	medium Dense	Dense	very Dense
Dr(%)	<15	15 to 35	35 to 65	65 to 85	85 to 100.

Determination of relative density :-

The below fig show the soil in the densest natural and loosest states. As it is difficult to measure the void ratio directly. However, it is convenient to express the void ratio in terms of dry density (ρ_d)



(a) (b) (c)

$$e = \frac{G\rho_w}{\rho_d} - 1$$

Representing the dry density in the loosest, densest and natural conditions as ρ_{min}, ρ_{max} and ρ_d

we know $D_r = \frac{e_{max} - e}{e_{max} - e_{min}} \times 100$

$$D_r = \frac{\left[\frac{G\rho_w}{\rho_{min}} - 1\right] - \left[\frac{G\rho_w}{\rho_d} - 1\right]}{\left[\frac{G\rho_w}{\rho_{min}} - 1\right] - \left[\frac{G\rho_w}{\rho_{max}} - 1\right]}$$

$$D_r = \frac{\rho_{max}}{\rho_d}\left[\frac{\rho_d - \rho_{min}}{\rho_{max} - \rho_{min}}\right]$$

Volumetric relationships :-

1. Void ratio (e) :-

It is defined as the ratio of the volume of voids to the volume of solids

$$e = \frac{V_V}{V_S} \longrightarrow (1)$$

It can be expressed by decimals such as 0.4, 0.5 for the coarse grained soil, the void ratio is generally smaller than that for fine-grained soils.

2. Porosity (n) :-

It is defined as the ratio of the volume of the voids to the total volume. Then

$$n = \frac{V_V}{V} \longrightarrow (2)$$

porosity generally expressed as percentage. However in equations it is used as a ratio. For example a porosity of 50% will be used as 0.5 in equations. The porosity can not exceed 100% as it would mean V_V is greater than V. Porosity is also known as percentage voids.

An inter-relationship can be found between the void ratio and the porosity as under from (2) $\frac{1}{n} = \frac{V}{V_V} = \frac{V_V + V_S}{V_V}$

$$\frac{1}{n} = 1 + \frac{1}{e} = \frac{1+e}{e} \longrightarrow (i)$$

$$n = \frac{e}{1+e} \longrightarrow (3)$$

from (i) $\frac{1}{e} = \frac{1}{n} - 1 = \frac{1-n}{n}$

$$e = \frac{n}{1-n} \longrightarrow (4)$$

In eqs (3) & (4) the porosity should be expressed as a ratio (and not percentage)

3. Degree of saturation (S) :-

The degree of saturation is the ratio of the volume of water to the volume of voids.

Thus $S = \frac{V_W}{V_V} \longrightarrow (5)$

The degree of saturation is generally expressed as a percentage. It is equal to zero when the soil is absolutely dry and 100% when the soil fully saturated. In expressions the degree of saturation is used as a decimal.

4. Percentage air voids (na):-

It is the ratio of the volume of air to the total volume

$$na = \frac{V_a}{V} \longrightarrow (6)$$

As the name indicates it reprasented as a percentage

5. Air content (ac):-

Air content is defined as the ratio of the volume of the air to the volume of voids.

$$ac = \frac{V_a}{V_v} \longrightarrow (7)$$

ac is usually expressed as a percentage.

Both air content and the percentage air voids are zero when the soil is saturated ($V_a = 0$)

An inter relationship between na & ac is

from (6) $na = \frac{V_a}{V} = \frac{V_a}{V_v} \times \frac{V_v}{V}$

$$\boxed{na = nac} \longrightarrow (8)$$

Volume - Mass Relationship :-

The volume - mass relationship are in terms of mass density. The mass of soil per unit volume is known as mass density. In soil engineering the following 5 different mass densities are used.

1. Bulk mass density :- The bulk mass density (ρ) is defined as the total mass (M) per unit total volume (V)

$$\rho = \frac{M}{V} \longrightarrow (1)$$

The bulk mass density is also known as the wet mass density (or) simply bulk density or density. It is expressed in kg/m^3, gm/ml (or) Mg/m^3

Obviously $1\,Mg/m^3 = 1000\,kg/m^3 = 1\,gm/ml$.

2. Dry mass density :-

It is defined as mass of solids per unit total volume.

$$\rho_d = \frac{M_s}{V} \longrightarrow (2)$$

The dry mass density is also known as dry density. The dry mass density is used to express the denseness of the soil. A high value of dry mass density indicates that the soil is in a compact condition.

3. <u>Saturated mass density :-</u>

The saturated mass density (ρ_{sat}) is the bulk mass density of the soil when it is fully saturated

$$\rho_{sat} = \frac{M_{sat}}{V} \longrightarrow (3)$$

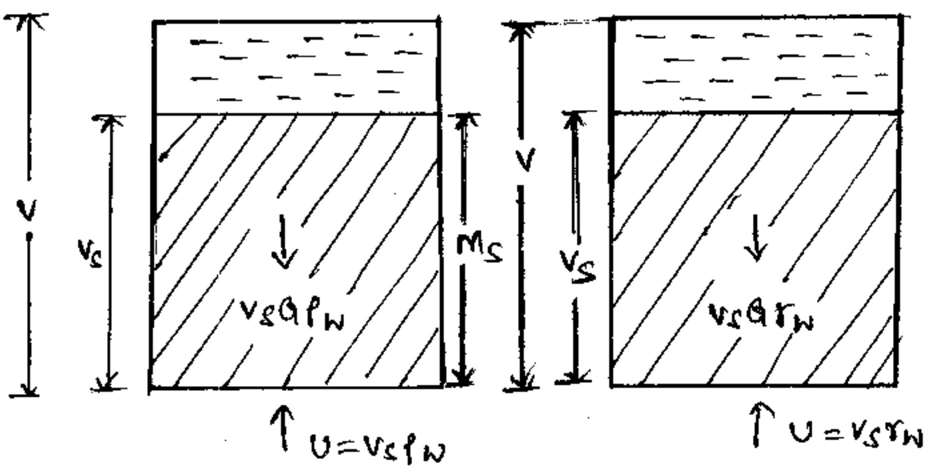
4. <u>Submerged Mass density :-</u>

When the soil exists below water it is in a submerged condition.

The submerged mass density (ρ') of the soil is defined as the submerged mass per unit of total volume

$$\rho' = \frac{M_{sub}}{V} \longrightarrow (4)$$

The submerged density is also expressed as ρ_{sub} in some texts. It is also known as the buoyant mass density (ρ_b).



(a) (b)

$$U = V_s \rho_w$$

$$M_{sub} = M_s - U$$

$$= V_s G \rho_w - V_s \rho_w$$

$$\rho' = \frac{V_s \rho_w (G-1)}{V}$$

Alternatively, we can also consider the equilibrium of the entire volume (V). In this case, the total downward mass, including the mass of voids, is given by

$$M_{sat} = M_s + V_v \rho_w$$

The total upward thrust, including that on the water in voids, is given by

$$U = V \rho_w$$

∴ The submerged mass is given by

$$M_{sub} = (M_s + V_v \rho_w) - V \rho_w$$

$$\rho' = \frac{(M_s + V_v \rho_w) - V \rho_w}{V} = \frac{M_{sat} - V \rho_w}{V}$$

$$= \frac{M_{sat}}{V} - \rho_w$$

$$\rho' = \rho_{sat} - \rho_w$$

Specific Gravity of solids :-

It is defined as the ratio of the unit weight of solids to the unit weight of water at a standard temparature at 4°c.

$$\boxed{G = \frac{\gamma_s}{\gamma_w}}$$

It is also defined as the ratio of mass of volume of solids to the mass of an equal volume of water at 4°c

$$\boxed{G = \frac{\rho_s}{\rho_w}}$$

Mass specific Gravity :-

It is defined as the ratio of mass (or) bulk unit weight of soil to the unit weight of water at standard temparature 4°c

$$\boxed{G_m = \frac{\gamma}{\gamma_w}}$$

It is also known as bulk specific gravity or apperent specific Gravity

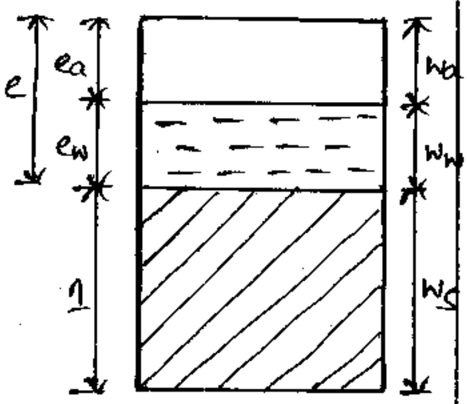
$$G > G_m$$

Three phase diagram in terms of void ratio :-

For convience the volume of solids is taken as unity.

We know $e = \frac{V_v}{V_s}$

$e = V_v \quad (\because V_s = 1)$



$\therefore$ total volume $(V) = 1+e$

Let volume of air $= e_a$

volume of water $= e_w$

Porosity $n = \frac{V_v}{V} = \frac{e}{1+e}$

Degree of saturation $S = \frac{V_w}{V_v} = \frac{e_w}{e}$

$$\boxed{e_w = Se} \longrightarrow (1)$$

$\therefore$ volume of air $V_a = e_a = e - e_w = e - Se \quad \{\because (1)\}$

$$\boxed{V_a = e(1-S)} \longrightarrow (2)$$

percentage of air voids $n_a = \frac{V_a}{V}$

$$n_a = \frac{e(1-S)}{1+e} \longrightarrow (3)$$

Air content $a_c = \frac{V_a}{V_v} = \frac{e(1-S)}{e}$

$$\boxed{a_c = 1-S} \longrightarrow (4)$$

Functional Relationships :-

i) Relationship between e, G, w and s :-

We know $s = \frac{V_w}{V_v} = \frac{e_w}{e}$ $\{\because V_v = e\}$

$$\boxed{e_w = se} \longrightarrow (1)$$

We know water content $w = \frac{W_w}{W_s}$

$$\gamma_w = \frac{W_w}{V_w}$$

$$W_w = V_w \gamma_w = se\gamma_w \quad \{V_w = e_w = se\}$$

$$\gamma_s = \frac{W_s}{V_s} \quad \{\because G = \frac{\gamma_s}{\gamma_w}\}$$

$$W_s = V_s \gamma_s = V_s G \gamma_w = G\gamma_w \quad \{\because \gamma_s = G\gamma_w\}$$

$$w = \frac{se\gamma_w}{G\gamma_w} \quad \{\because V_s = 1\}$$

$$w = \frac{se}{G}$$

$$\boxed{se = wG} \longrightarrow (2)$$

2. Relation between γ_d, G & e (or) n :-

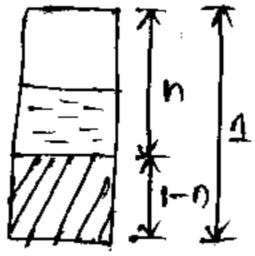
We know $\gamma_d = \frac{W_s}{V}$

$$\gamma_d = \frac{\gamma_s V_s}{V} \quad \{\because \gamma_s = \frac{W_s}{V_s}\}$$

$$\gamma_d = \frac{\gamma_s}{1+e} \quad \{\because V = 1+e \text{ from fig}\}$$

but we know $\gamma_s = G\gamma_w$

$$\boxed{\gamma_d = \frac{G\gamma_w}{1+e}} \longrightarrow (3)$$



$$\boxed{e = \frac{G\gamma_w}{\gamma_d} - 1} \longrightarrow (4)$$

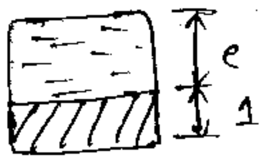
In case of n

$$\gamma_d = \frac{W_s}{V} = \frac{\gamma_s V_s}{V} = \frac{G\gamma_w(1-n)}{1}$$

$$\boxed{\therefore \gamma_d = G\gamma_w(1-n)} \longrightarrow (5)$$

3. <u>Relation between γ_{sat}, G and e :-</u>

we know $\gamma_{sat} = \frac{W_{sat}}{V}$



$$= \frac{W_w + W_s}{V}$$

$$= \frac{\gamma_w V_w + \gamma_s V_s}{V}$$

$\cancel{\frac{\gamma_s V_s}{V} = \gamma_d}$

$$= \frac{\gamma_w e + G\gamma_w}{1+e}$$

$$\left\{ \begin{aligned} \gamma_w &= \frac{W_w}{V_w} \\ W_w &= \gamma_w V_w \\ \gamma_s &= \frac{W_s}{V_s} \\ W_s &= \gamma_s V_s \end{aligned} \right\}$$

$$\boxed{\gamma_{sat} = \frac{(G+e)\gamma_w}{1+e}} \longrightarrow (6)$$

4. <u>Relation between γ, G, e and S :-</u>

we know $\gamma = \frac{W}{V}$

$$= \frac{W_w + W_s}{V}$$

$$\gamma = \frac{\gamma_w V_w + \gamma_s V_s}{V}$$ $\therefore \left\{ \begin{array}{l} V_w = e_w = es \\ V_s = e_s = 1 \end{array} \right\}$

$$= \frac{\gamma_w (es) + G\gamma_w (1)}{V}$$ $\left\{ \begin{array}{l} G = \frac{\gamma_s}{\gamma_w} \\ V_s = G\gamma_w \end{array} \right\}$

$$\boxed{\gamma = \frac{\gamma_w (es+G)}{1+e}} \longrightarrow (7)$$

5. <u>Relationship between γ', G and e</u>

We know $\gamma' = \gamma_{sat} - \gamma_w$

$$= \frac{(G+e)\gamma_w}{1+e} - \gamma_w$$

$$= \gamma_w \left[\frac{G+e-1-e}{1+e} \right]$$

$$\boxed{\gamma' = \frac{(G-1)\gamma_w}{1+e}} \longrightarrow (8)$$

6. <u>Relation between γ_d, γ and w :-</u>

We know $w = \frac{W_w}{W_s}$

Both sides add '1'

$$1+w = \frac{W_s + W_w}{W_s}$$

$$= \frac{W}{W_s}$$

$$W_s = \frac{W}{1+w}$$

$$\gamma_d = \frac{W_s}{V} = \frac{W}{(1+w)} \times \frac{1}{V} = \frac{\gamma}{1+w}$$ $\left\{ \because \frac{W}{V} = \gamma \right\}$

$$\boxed{\gamma_d = \frac{\gamma}{1+w}} \longrightarrow (9)$$

7. Relationship between γ_d, G, w and S :-

$$\gamma_d = \frac{G\gamma_w}{1+e}$$

We know $Se = wG$

$$e = \frac{wG}{S}$$

$$\boxed{\therefore\ \gamma_d = \frac{G\gamma_w}{1+\frac{wG}{S}}} \longrightarrow (10)$$

8. Relation between γ_{sat}, γ, γ_d and S :-

We prove

$$\gamma = \frac{(G+Se)\gamma_w}{1+e} \qquad \left\{\because \text{from eq } (7)\right\}$$

$$= \frac{G\gamma_w}{1+e} + \frac{Se\gamma_w}{1+e}$$

$$= \gamma_d + S\left[\frac{(G+e)\gamma_w}{1+e} - \frac{G\gamma_w}{1+e}\right]$$

$$\boxed{\gamma = \gamma_d + S[\gamma_{sat} - \gamma_d]} \longrightarrow (11)$$

9. Relation between γ_d, G, w, n_a :-

We know

$$V = V_a + V_w + V_s$$

$$= V_a + \frac{W_s}{\gamma_s} + \frac{W_w}{\gamma_w} \qquad \left\{\gamma_s = \frac{W_s}{V_s} \quad \gamma_w = \frac{W_w}{V_w}\right\}$$

dividing both sides with 'V'

$$\frac{V}{V} = \frac{V_a}{V} + \frac{W_w}{V\gamma_w} + \frac{W_s}{V\gamma_s}$$

$$1-\frac{V_a}{V} = \frac{w \cdot W_s}{V \cdot \gamma_w} + \frac{W_s}{V\gamma_s}$$

$$1-n_a = \frac{\gamma_d w}{\gamma_w} + \frac{\gamma_d}{\gamma_s}$$

$$1-n_a = \frac{\gamma_d}{\gamma_w}\left(w+\frac{1}{G}\right)$$

$$\frac{(1-n_a)\gamma_w}{w+\frac{1}{G}} = \gamma_d$$

$$\boxed{\gamma_d = \frac{(1-n_a)G\gamma_w}{wG+1}} \longrightarrow (12)$$

List of formulaes :-

1. $n = \frac{e}{1+e}$

2. $e = \frac{n}{1-n}$

3. $n_a = n a_c$

4. $\gamma = \frac{(G+Se)\gamma_w}{1+e}$

5. $\gamma_d = \frac{G\gamma_w}{1+e}$

6. $\gamma_{sat} = \frac{\gamma_w(G+e)}{1+e}$

7. $\gamma' = \frac{(G-1)\gamma_w}{1+e}$

8. $Se = wG$

9. $\gamma_d = \dfrac{\gamma}{1+w}$

10. $\gamma_d = \dfrac{(1-n_a)\,G\gamma_w}{1+wG}$

11. $\gamma = \dfrac{(G+Se)\gamma_w}{1+e} = \dfrac{(G+wG)\gamma_w}{1+e} = \dfrac{G\gamma_w(1+w)}{1+\frac{wG}{S}}$

12. If soil is saturated $S=1$

$$\gamma_{sat} = \frac{G\gamma_w(1+w)}{1+wG}$$

$$\gamma_{sat} = \frac{G\gamma_w(1+w)}{1+e}$$

13. $V_v = n$

14. $V_s = 1-n$

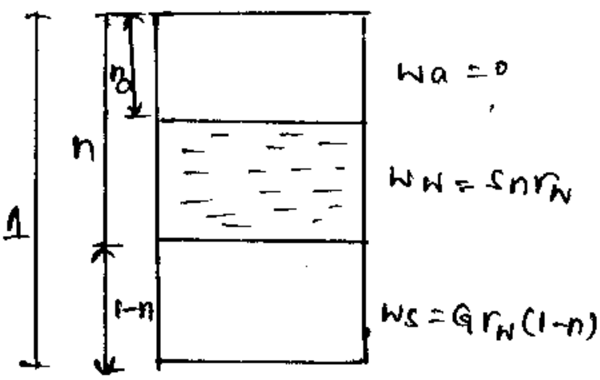
15. $e = \dfrac{n}{1-n}$

16. $\gamma = \gamma_w\,(G(1-n)+Sn)$

17. $\gamma_d = G\gamma_w(1-n)$

18. $\gamma_{sat} = [G(1-n)+n]\,\gamma_w$

19. $\gamma' = [(G-1)(1-n)]\,\gamma_w$



Probleme :-

1. Determine water content, Dry density, bulk density, void ratio and degree of saturation from the following data. sample size 3.81 cm ϕ x 7.62 cm ht, wet weight 1.668N oven dry weight = 1.400N & sp. gravity (G) is 2.7.

Sol:- water content $w = \frac{W_w}{W_s} = \frac{1.668 - 1.400}{1.4} \times 100$

$= \frac{0.268}{1.4} \times 100$

$= 19.14\%$

volume of the sample $V = A \times h$

$= \frac{\pi}{4} \times 3.81^2 \times 7.62$

$= 86.88\ cm^3$

$\gamma_w = 1 g/c$
$= 1000\ kg/m^3$
$= 9.8\ kN/m^3$
$\gamma_w = 9810 N/m^3$

Dry density $(\gamma_d) = \frac{W_s}{V} = \frac{1.4}{86.88} = 0.01611$

$= 16.11\ kN/m^3$

Bulk unit weight $(\gamma) = \frac{W}{V} = \frac{1.668}{86.88}$

$= 0.0192\ N/cm^3$

$= 19.2 N/m^3$

$\gamma_d = \frac{G\gamma_w}{1+e} = \frac{2.7 \times 9.81}{1+e}$

$16.11 = \frac{2.7 \times 9.81}{1+e}$

$1+e = 1.644$

$e = 0.645$

Degree of saturation (s)

We know $Se = wG$

$$S = \frac{0.1914 \times 2.7}{0.645} \times 100$$

$$S = 80.12\%$$

8. A partially saturated soil sample obtained from a earth fill has a natural moisture content of 22% and unit weight of 19.62 kN/m^3 of $G = 2.7$ and unit weight of water $= 9810\ N/m^3$ compute

a) Degree of saturation b) void ratio

c) If subsequently the soil gets saturated, find its unit weight.

Sol:- $w = 22\%$ $\gamma = 19.62\ kN/m^3$ $G = 2.7$

$\gamma_w = 9810\ N/m^3 = 9.81\ kN/m^3$

$$\gamma = \frac{G\gamma_w(1+w)}{1+e} = \frac{2.7 \times 9.81(1+0.22)}{1+e}$$

$$1+e = \frac{26.487(1.22)}{19.62}$$

$$1+e = 1.647$$

$$e = 0.647$$

$Se = wG$

$$S = \frac{0.22 \times 2.7}{0.647} = 0.9188 = 91.88\%$$

Soil gets saturated then $S = 1$

$$\gamma_{sat} = \frac{(G+e)\gamma_w}{1+e} = \frac{(2.7+0.647) \times 9.81}{1+0.647} = 19.92\ kN/m^3$$

5. A sample of saturated soil has a water content of 38%. $G = 2.65$. Determine void ratio, porosity, saturated unit weight and dry unit weight.

Given sample is saturated than $S = 1$

$$w = 38\%$$

$$G = 2.65$$

$$Se = wG$$

$$e = \frac{wG}{S}$$

$$= 0.38 \times 2.65 = 1.007$$

Porosity $n = \frac{e}{1+e}$

$$= \frac{1.007}{1+1.007} = 0.5017$$

$$= 50.17\%$$

Taking $\gamma_w = 9.81\ kN/m^3$

$$\gamma_{sat} = \frac{(G+e)\gamma_w}{1+e}$$

$$= \frac{(2.65+1.007)9.81}{1+1.007}$$

$$= 17.88\ kN/m^3$$

$$\gamma_d = \frac{G\gamma_w}{1+e} = \frac{2.65 \times 9.81}{1+1.007} = 12.952\ kN/m^3$$

4. A sand sample has porosity of 28% and specific gravity of solids 2.65 find

a) Dry unit weight of sand b) unit weight of sand if $S = 0.56$

c) unit wt of saturated sand.

d) U.W in submerged condition

Sol:

$n = 28\%$, $G = 2.65$

$$e = \frac{n}{1-n} = \frac{0.28}{1-0.28} = 0.388$$

a) $$\gamma_d = \frac{G\gamma_w}{1+e} = \frac{2.65 \times 9.81}{1+0.388} = \frac{25.9965}{1.388} = 18.729 \text{ kN/m}^3$$

b) $S = 0.56$

$$\gamma = \frac{(G+Se)\gamma_w}{1+e} = \frac{(2.65+0.56\times0.388)\times9.81}{1+0.388}$$

$$= \frac{28.128}{1.388} = 20.2651 \text{ kN/m}^3$$

c) $S = 1$

$$\gamma_{sat} = \frac{(G+e)\gamma_w}{1+e} = \frac{(2.65+0.388)\times9.81}{1+0.388} = 21.471 \text{ kN/m}^3$$

$$Se = wG \Rightarrow w = \frac{Se}{G} = \frac{0.56\times0.388}{2.65} = 0.081$$
$$= 8.1\%$$

d) unit weight in submerged condition

$$\gamma' = \frac{(G-1)\gamma_w}{1+e} = \frac{(2.65-1)\times9.81}{1+0.388} = 11.661 \text{ kN/m}^3$$

(or)

$$\gamma' = \gamma_{sat} - \gamma_w$$
$$= 21.471 - 9.81 = 11.661 \text{ kN/m}^3$$

5. A saturated clay has a water content 39.3% and a bulk sp. gravity of 1.84 final void ratio and sp. gravity of particles.

Sol:

$w = 39.3\%$ $\quad G_m = 1.84$

We know $G_m = \frac{\gamma}{\gamma_w}$

$\gamma = G_m \cdot \gamma_w$

$= 1.84 \times 9.81 = 18.05 \text{ kN/m}^3$

(or)

$\gamma = 1.84 \times 1 = 1.84 \text{ kN/m}^3$ $\quad (\because \gamma_w = 1 \text{ g/cc})$

since it is saturated than $\gamma = \gamma_{sat}$

$= 1.84 \text{ g/cc}$

$Se = wG$

$e = wG$

$$\gamma_{sat} = \frac{(G+e)\gamma_w}{1+e} = \frac{(G+wG)\gamma_w}{1+e}$$

$$1.84 = \frac{G(1+0.393)\times 1}{1+0.393\times G}$$

$$= \frac{1.393G}{1+0.393G}$$

$1+0.393G = 0.757G$

$0.364G = 1.$

$$\boxed{G = 2.747}$$

$e = wG$

$= 0.393 \times 2.747$

$e = 1.0795$ //

6. A soil has porosity of 40% and G = 2.65 and water content 12%. determine mass of water to added to 100 m^3 of this soil for full saturated.

Sol:

$n = 90\%$ $\quad w = 12\%$ $\quad G = 2.65$

let us take volume of solids = 1 m^3

wt of solids $(W_s) = V_s \gamma_s = G \gamma_w V_s$

$= 2.65 \times 1000 \times 1$

$= 2650$ kg

$\therefore$ wt of water $= W_w = w \times W_s$

$= 0.12 \times 2650 = 318$ kg

volume of water $V_w = \frac{318}{1000} = 0.318\ m^3$

$e = \frac{n}{1-n} = \frac{0.4}{1-0.4} = 0.667$

$e = \frac{V_v}{V_s}$

$V_v = e \times V_s = 0.667 \times 1 = 0.667\ m^3$

volume of air $V_a = V_v - V_w = 0.349\ m^3$

volume of addition water for full saturation $= 0.349\ m^3$

$\therefore$ Total volume $V = V_w + V_s + V_a$

$= 0.318 + 1 + 0.349 = 1.667\ m^3$

$1.667 \rightarrow 0.349$

$100 m^3 \rightarrow \frac{100 \times 0.349}{1.667} = 20.935\ m^3$

volume of water required for 100 m^3 of soil

$= 20.935\ m^3$

$= 20935$ kg

The properties of Soils which are not of primary interest to the geotechnical engineer but which are indicative of the engineering properties are Called "index properties". Simple test which are required to determine the index properties are known as Classification test.

the index properties are Some times divided into two categories.

1. properties of individual particles
2. properties of Soil mass also known as aggregate properties.

Moisture Content :

The moisture Content (m) is defined as the ratio of the mass of water to the mass of Solids.

$$m = \frac{M_w}{M_s} \text{ (or) } \frac{W_w}{W_s}$$

The moisture content is also known as the water Content (w). It Can be expressed as percentage, but used as a decimal in Computation.

The water Content of the fine-grained Soils, Such as Silts and clays is generally more than Coarse grained Soils.

In geology and Some other disciplines, the water Content is defined as the ratio of the mass water to total mass. Some instruments such as moisture tester, also give the water Content as a ratio of the total mass.

$$m' = \frac{M_w}{M} \times 100$$

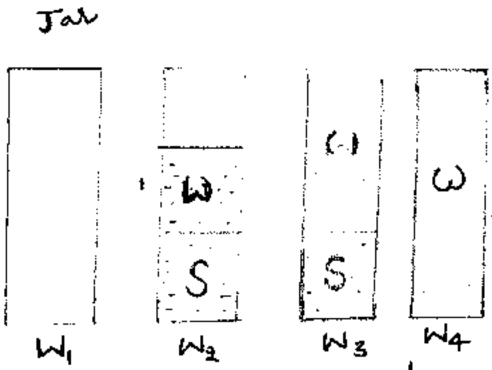
Determination of Water Content :

1. Oven dry method.
2. pyconometer method.
3. Calcium Carbide method
4. Alcohol method
5. Torsion balance method
6. Sand bath method
7. Radiation method

1. Oven dry method :

First take the wet Sample and weigh it and put it in oven upto 24 hours and again weigh it. It gives the dry weight. By using $w = \frac{W_w}{W_s}$ we can find out the water Content. If the time was less the minimum time that the Soil is be in the oven is for clayee Soil minimum 15 hours and for Sandy Soil minimum 4 hours and the maximum in 24 hours.

(2) Pyconometer method :-



The Jar was taken ded and weighted (W_1) and the moist Soil was taken in the jar and again weigh it (W_2) and the remaining space will be filled with water and weigh it (W_3). The Soil water is removed and and the jar was filled with water and weighed (W_4).

and the formula is $W = \left[\frac{W_2 - W_1}{W_3 - W_4} \left(\frac{G-1}{G} \right) - 1 \right] \times 100$

Volume of Solids $= \frac{W_s}{G}$

$$\left[\begin{aligned} W_s &= V_s \gamma_s \\ &= V_s G \gamma_w \\ &= V_s G (1) \\ W_s &= G V_s \\ V_s &= \frac{W_s}{G} \end{aligned} \right]$$

$$W_4 = W_3 - W_s + \frac{W_s}{G}$$

$$W_3 - W_4 = W_s \left(1 - \frac{1}{G} \right)$$

$$W_s = (W_3 - W_4) \left(\frac{G}{G-1} \right)$$

Weight of water in Soil Sample $W_w = (W_2 - W_1) - W_s$

we know water Content $= \frac{W_w}{W_s}$

$$\omega = \frac{(\omega_2 - \omega_1) - \omega_s}{\omega_s} = \frac{\omega_2 - \omega_1}{\omega_s} - 1$$

$$\omega = \left[\frac{\omega_2 - \omega_1}{\omega_3 - \omega_4} \left(\frac{G-1}{G} \right) - 1 \right]$$

③. Calcium Carbide method :-

In this method there is a Cylinder which has reading in above the Cylinder. the soil taken and calcium Carbide also taken which is equal to the Soil and mixed then thoroughly and inserted into the Cylinder. Cylinder was shaked upto 5 to 10 min. the water which the Soil is having is reacted with $CaCO_3$ and provide acetylene gas (C_2H_2) for the pressure of the gas. the Readings will be changed in the meter. It will shows the water Content of the Soil.

$$CaC_2 + 2H_2O \longrightarrow C_2H_2 + Ca(OH)_2$$

Specific gravity :-

The Specific gravity of Solid particles (G) is defined as the ratio of the mass of a given volume of Solids to the mass of an equal volume of water at 4°c. Thus, the Specific gravity is given by

$$G = \frac{\rho_s}{\rho_w}$$

The mass density of water ρ_w at 4°c is one gm/mL, 1000 kg/m³ (or) 1 mg/m³

The Specific gravity of Solids for most natural soil falls in the general range of 2.65 to 2.80.

Specific gravity of Solids is an important parameter. It is used for determination of Void ratio and particle Size.

Typical Values of Sp. gravity.

1. Gravel	—	2.65 – 2.68
2. Sand	—	2.65 – 2.68
3. Silty Sands	—	2.66 – 2.70
4. Inorganic Clays	—	2.68 – 2.80
5. Silt	—	2.66 – 2.70
6. organic Soils	—	Variable may fall below 2.00

In addition to the Standard term of Specific gravity as defined, the following two terms related with the Specific gravity are also occasionally used.

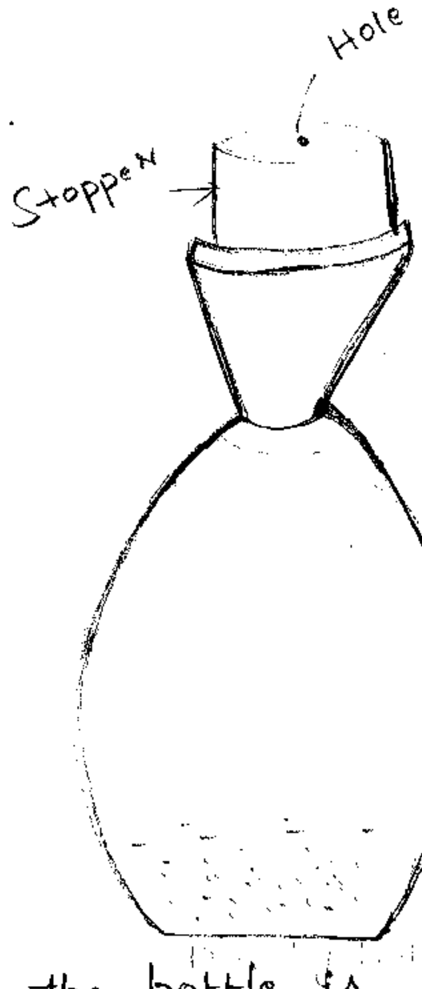
1. Mass Specific gravity (G_m)
2. Absolute Specific gravity (G_a)

→ Refer I-unit ←

Specific gravity determination

The Specific gravity of Solid particles is determined in the laboratory using the following methods.

1. Density bottle method.
2. Gas jar method.
3. pyconometer method.
4. measuring flask method.
5. shrinkage limit method.



1. Density Bottle method :-

(i) Take a clean and dry density bottle and weigh it with stopper. let the weight be W_1.

(ii) Take about 10-20 gm of an oven dried Soil Sample into it and fixed the weight of the bottle and the Soil with stopper, let it be W_2.

(iii) Add distilled water so that the bottle is half full, remove the entrapped air by Connecting it to Vaccum source.

(iv) Fill the bottle Completely with distilled water, put the stopper and wipe it clean. Determine the weight of the bottle and its Content 'W_3'

(V) Empty the bottle and clean it thoroughly. Fill it with distilled water, put the stopper and wipe ~~the~~ the bottle dry on outside. Find its weight 'W_4'

(VI) Repeat the steps 2 to 5 on more samples of the given soil and find the result by using below formulae.

$$\text{Specific gravity } G = \frac{W_2 - W_1}{(W_2 - W_1) - (W_3 - W_4)}$$

W_1 = weight of empty density bottle with stopper

W_2 = weight of bottle with stopper + dry soil

W_3 = weight of bottle with stopper + soil + water

W_4 = weight of bottle with stopper + water

2. Pyconometer method:-

This method is similar to the density bottle method. As the capacity of the pyconometer is larger about 200 - 300g of oven-dry soil is required for the test. The method can be used for all types of soils, but is more suitable for medium grained soils, with more than 90% passing a 20mm IS sieve and for coarse grained soils with more than 90% passing a 40mm IS sieve.

3. Measuring Flask method :-

A measuring flask is of 250 ml (or) (500 ml) Capacity. with a graduation mark at the level. It is fitted with an adapter for Connecting it to a vaccum line for removing entrapped air. This method is similar to the density bottle method. About 80 to 100 g of oven dry Soil is required in this case. this method is Suitable for fine grained and medium grained Soils.

4. Gas jar method :-

In this method, a gas jar of about 1 Lt Capacity is used. The jar is fitted with a rubber bung. the gas jar Serve as a pyconometer. This method is Similar to the pyconometer method.

CONSISTENCY LIMITS

"The water Contents at which the soil changes from one state to the other are known as Consistency limits or Atterberg's Limits"

We have three types of Consistency limits. They are

1. Liquid limit
2. plastic limit
3. shrinkage limit

1. Liquid Limit :-

"The liquid limit is the water Content at which the Soil changes from the liquid state to plastic state.

At the liquid state, the Clay is practically Like a liquid, but possesses a Small shearing strength. the liquid limit of Soil depends upon the clay mineral present in the Soil.

The liquid limit is determined in the laboratory either by Casagrande's apparatus or by cone-penetration method. the device used in the Casagrande method Consists of a brass cup which drops through a height of 1 cm from the hard base.

Procedure :-

1. Adjust the cup of the liquid limit apparatus with the help of grooving tool gauge and adjustment plate to give a drop of exactly 1 cm on the point of Contact on base.

2. Take about 120 gms of air-dried Sample passing 425 μ Sieve.

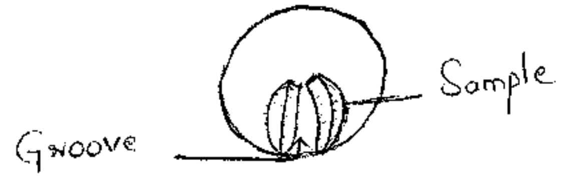
3. Mix it thoroughly with quantity of distilled water to form a uniform paste.

4. place a portion of the paste in the Cup. Smooth the Surface with Spatula to a minimum depth of 1 cm. Draw grooving tool through the Sample along the Symmetrical axis of the cup. holding the tool perpendicular to the cup.

5. Turn the handle at a rate of 2 revolutions per Second and Count blows untill the two parts of the Sample come in Contact at the bottom of the groove.

6. Transfer the remaining Soil in the Cup to the main Soil Sample and mix thoroughly after adding a small amount of water.

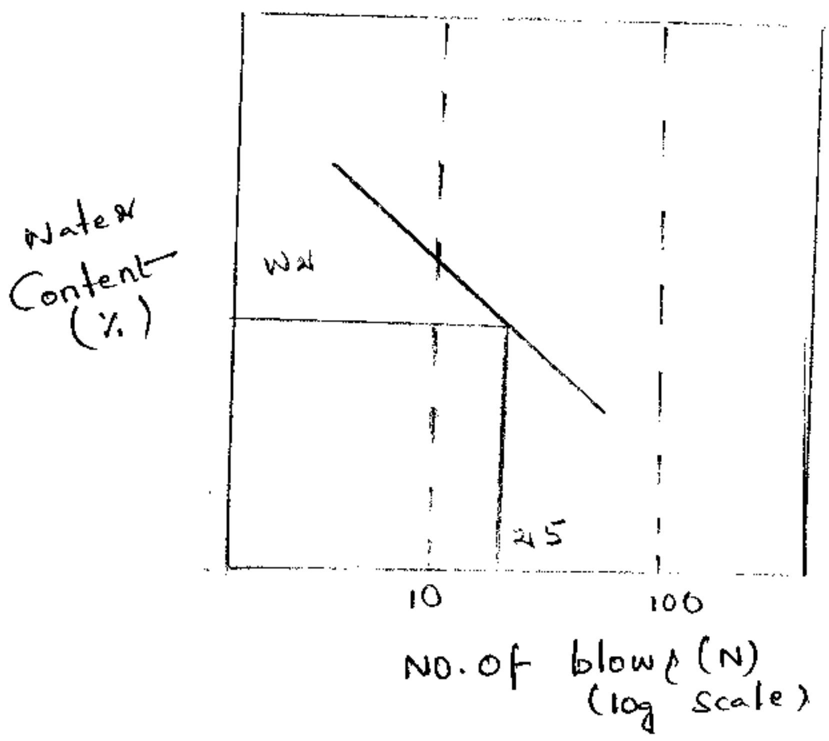
7. Repeat the steps 4, 5 and 6. Obtain at least five sets of reading in the range of 10 to 50 blows.



Exp:-

S.NO	Amount of water added (ml)	Moisture Content (%)	No. of blows.
1	30	30%	4
2	29	29%	10
3	28	28%	15

plot a Straight line graph between no. of blows and water Content. Read the water Content at 25 blows, which is the value of liquid limit.



2. plastic limit :-

plastic limit is the water Content below which the Soil stops behaving as a plastic material

(or)

The moisture Content at which soil has the Smallest plasticity is Called the plastic limit.

Procedure :-

1. Take about 30 gm of air dried Sample passing Though 425 micron Sieve.
2. Mix thoroughly with distilled water on the glass plate until it is plastic enough to be shaped into Small ball.
3. Take about 10 gm of the plastic Soil mass and roll it between the hand and the glass-plate to form the Soil mass into a thread. if the diameter of thread becomes less than 3 mm without cracks, shows that water is more than its plastic plastic limit, hence the Soil is kneaded further and rolled into thread again.
4. Repeat this rolling and remoulding process untill the the thread Starts just Crumbling at the diameter of 3 mm.
5. If Crumbling Starts before 3 mm diameter thread, it shows that water added is less than the plastic limit of the Soil, hence Some more

water should be added and mixed to a uniform mass and rolled again, until the thread starts crumbling at a diameter of 3mm.

6. Collect the pieces of crumbled soil thread at 3 mm diameter in an air tight container and determine moisture content

7. Repeat this procedure for two more samples.

SHRINKAGE LIMIT:

Shrinkage limit is defined as the maximum water content at which a reduction in water content will not cause a decrease in the volume of a soil mass. It is the lowest water content at which a soil can still be completely saturated.

$$\text{Shrinkage limit } W_s = \left[W_1 - \frac{V - V_d}{W_d}\right]$$

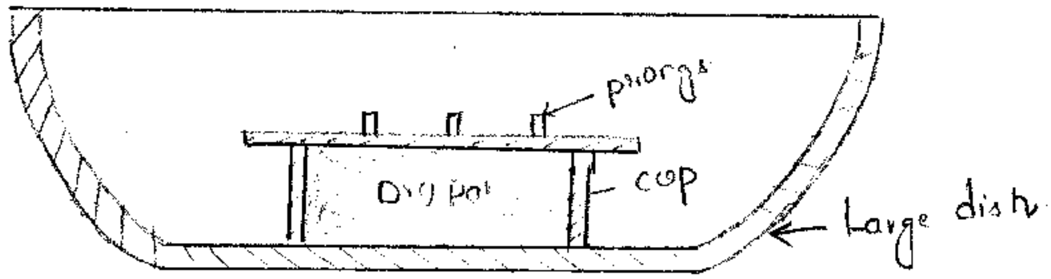
Where

W_1 = moisture content

V = Volume of wet soil = volume of dry soil

V_d = Volume of dry soil pat

W_d = weight of the dry soil pat.



Procedure for Shrinkage limit test:-

1. Take about 100 gm of Soil Sample passing through 425 μ IS Sieve.
2. place about 30gm of the Soil in evapourating dish and mix it thoroughly with distilled water such that water added will Completely fill the Voids in the Soil and make the Soil pasty enough to be readily worked out into the shrinkage dish without entrapping air.
3. Weigh a clean and dry Shrinkage dish.
4. place the Shrinkage dish in Evapourating dish fill it with mercury. Remove the excess mercury, clean the dish and find the weight of mercury in the shrinkage dish. Volume of shrinkage dish will be obtained by dividing the weight of mercury by its unit weight volume of the wet Soil pat will be equal to the Volume of shrinkage dish.

5. Apply a thin Coat of grease on the inside of the shrinkage dish.

6. place the Soil paste at the Centre of the dish and tap it on firm Surface and allow the paste to flow towards edges. Continue the tapping till the Soil is Compacted. and entrapped air is removed. Repeat the process till the dish is Completly filled with Soil.

7. Weigh the shrinkage dish with wet Soil.

8. Keep the dish in air till the Colour turns from dark to light and there keep it in oven for 24 hours at Constant temparature of 105°c

9. cool the dish and weigh it immediately.

10. Determine the Volume of dry Soil pat by immersing it in mercury and measuring the volume of mercury displaced.

11. Repeat the procedure for two more Samples.

1. Plasticity Index :- (I_p (or) PI)

I_p is the range of water Content over which the Soil remains in the plastic State. It is equal to the differance between the liquid limit (w_l) and plastic limit (w_p).

$$\boxed{I_p = w_l - w_p}$$

if the plastic limit is greater than the liquid limit. the plasticity index is reported at zero (and not $-$Ve)

2. Liquidity Index :-

Liquidity index (I_l or LI) is defined as.

$$I_l = \frac{w - w_p}{I_p} \times 100$$

Where. w = Water Content of the Soil in natural Condition.

The Liquidity index of a Soil indicates the nearness of its water Content to its liquid limit. when the Soil is liquid limit, its liquidity index is 100%. and it becomes as a liquid. when the Soil is and behaves at the plastic limit. its liquidity index is zero.

Consistency Index :-

$$I_c = \frac{w_l - w}{I_p} \times 100$$

4. Shrinkage Index :- The Shrinkage index (I_s) is the numerical differance between the liquid limit (W_L) and the shrinkage limit (W_s).

$$\boxed{I_s = W_l - W_s}$$

5. Shrinkage Ratio :-

The Shrinkage Ratio (SR) is defined as the ratio of given volume change, expressed as the percentage of dry volume to the Corresponding change in water Content.

$$SR = \frac{(V_1 - V_2) * V_d}{(W_1 - W_2)} \times 100$$

Where, V_1, V_2 = Volume of Soil at W_1, W_2.

V_d = Volume of dry Soil mass.

6. Volumetric Shrinkage :-

The Volumetric Shrinkage (VS) (or) Volumetric change is defined as the change in volume expressed as percentage of the dry volume. when the water content is reduced from a given value to the shrinkage limit. That.

$$V_s = \left[\frac{V_1 - V_d}{V_d}\right] * 100.$$

Indian standard Classification of Soils :-

The System uses particle Size analysis and plasticity chart for the classification of the Soil. in the System the Soils are Classified into 18 groups.

The Soil first Classified into three Categories.

(1) Coarse grained Soils
(2) Fine - grained Soils
(3) High organic Soils (peat)

(1) Coarse grained Soils :-

Coarse grained Soils are Sub-divided into gravel and sand. The Soil is termed gravel (CG) when more than 50% of Coarse fraction is retained on 4.75 mm IS Sieve and termed sand (S), if more than 50% of the Coarse fraction is smaller than 4.75 mm. IS Sieve. Coarse grained Soils are further sub-divided as given as the table no: 01.

(2) Fine grained Soils :- the fine grained Soils are further divided into three Sub-divisions, depending upon the Values of the liquid limit.

(a) silts and clays of low Compressibility :- These soils have a liquid limit less than 35

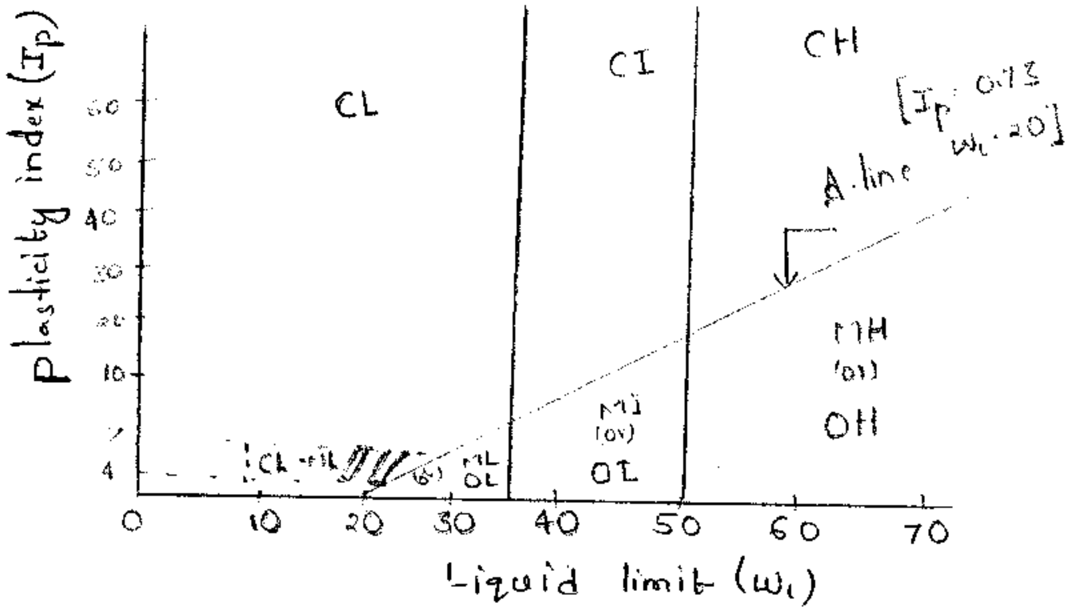
(b). silts and clays of Medium Compressibility:-
These Soils have a liquid limit greater than 35 but less than 50 (symbol is I)

(c). Silts and Clays of high Compressibility:-
These Soils have a liquid limit greater than 50.

Fine-grained Soils are further Sub-divided into 9 groups in table no: 02.

(3). High organic Soils:-
If the Soil is highly organic and Contains a large percentage of organic matter and particles of decomposed Vegetation. it is kept in a Seperate Category marked peat (Pt)

Plasticity Chart



problems

1. Determine the liquid limit, plasticity index, liquidity index from following data.

Water Content (w)	55	46	32	22	15
No. of blows (N)	24	30	35	41	49

The plastic limit is 24% and Natural water Content is 32%

$w_p = 24\%$

$w = 32\%$

liquid limit = 53.5%

plasticity index = Liquid limit − plastic limit

$= 53.5 - 24$

$= 29.5\%$

liquidity index $= I_L = \dfrac{w - w_p}{I_p} \times 100$

$= \dfrac{32 - 24}{29.5} \times 100$

$= 27.1\%$

2. An undisturbed Saturated Specimen of clay has a volume of 18.9 cm^3 and a mass of 30.2 gm on oven drying the mass reduces to 18 gm. The volume of dry specimen is 9.9 cm^3. Determine shrinkage limit, Specific gravity of Solids, shrinkage ratio and volumetric shrinkage?

$W_1 = 30.2\ gm$, $W_s = 18\ gm$

V_d (or) $V_2 = 9.9\ cm^3$ $\quad V_1 = 18.9\ cm^3$

$\gamma_w = 1\ g/cc$

$$\omega_s = \left[\omega - \frac{(V_1 - V_2)\gamma_w}{W_s} \right] \times 100 \qquad \left\{ \omega = \frac{W_1 - W_s}{W_s} \times 100 \right\}$$

$$= \left[\frac{W_1 - W_s}{W_s} - \frac{(V_1 - V_2)\gamma_w}{W_s} \right] \times 100$$

$$= \left[\frac{30.2 - 18}{18} - \frac{(18.9 - 9.97) * 1}{18} \right] \times 100$$

$$= \left[0.678 - 0.5 \right] \times 100$$

$$= 17.8\%$$

$$G = \frac{W_s}{V_1\gamma_w - (W_1 - W_s)} = \frac{18}{18.9 \times 1 - (30.2 - 18)} = 2.69$$

$$S.R = \frac{\gamma_d}{\gamma_w} = \frac{W_s}{V\gamma_w} = \frac{18}{9.9 \times 1} = 1.818$$

$$V_s = \frac{V_1 - V_d}{V_d} \times 100 = \frac{18.9 - 9.9}{9.9} \times 100 = 91\%$$

(or) $\qquad \left\{ V_d = \frac{W_s}{V} \right\}$

$$V_s = SR / (W_1 - W_s)$$

$$= (67.8 - 17.8) * 1.82$$

$$= 91\%$$

UNIT - III

PERMEABILITY

Soil Water:-

Water present in the voids of soil mass is called Soil water.

The soil water is brodly classified into two categories

1. free water.
2. Held water.

free water moves in the pores of the soil under the influence of gravity. The held water is reteined in the pores of the soil, and it can not move under the influence of gravitational force.

free water flows from one point to the other wherever there is a difference of total head.

Held water is further divided in to three types.

(i) Structural Water:- The structural water is chemically combined water in the crystel structure of the mineral of the soil. This water can not be removed without breaking the structure of the mineral.

(ii) Adsorbed Water:- The water held by electrochemical forces existing on the soil surface is known as adsorbed water (or) hygroscopic water.

(iii) Capillary Water:- The water held in the interstices of soil due to capillary forces is called cappillary water.

Capillary rise in soils :-

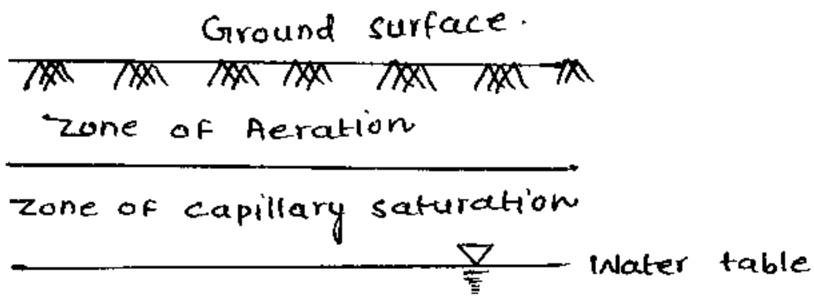
capillary rise in soils depends upon the size and grading of the particles. The diameter (d) of channel in pore passage depends upon the diameter of the particle. it is generaly taken as 1/5 th diameter of the effective diameter (D_{10}) in the case of coarse - grained.

$$\text{Thus } d = 0.2 D_{10}$$

The space above the water table can be devided in to two regions.

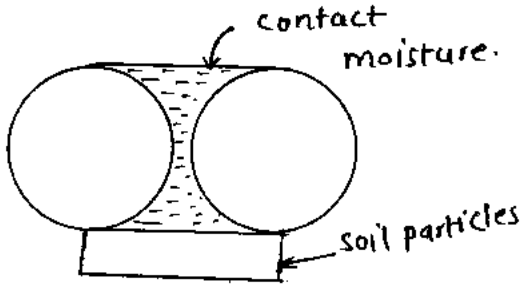
1 Zone of capillary saturation

2 Zone of Aeration.



* In zone of capillary saturation the soil is fully saturated
* In zone of Aeration the soil is not saturation.
* The heigh to which capillary water rises in soils is known as "capillary fringe".
* The soil above the capillary fringe may contain water in the form of contact water.

* Terzaghi and pick (1948) gave a relation between The maximum height of capillary fringe and the effective size as.



$$(h_c)_{max} = \frac{C}{e D_{10}}$$

Where C = constant, depending upon the shape of the grain and impurities

e = void ratio.

D_{10} = effective diameter, the size corresponding to 10 % finer.

if D_{10} is in mm, the value of varies between 10 to 50 mm^2, and the height $(h)_{max}$ is also given in mm. if D_{10} and $(h_c)_{max}$ are in centimeters,

$C = 0.1$ to $0.5\ cm^2$

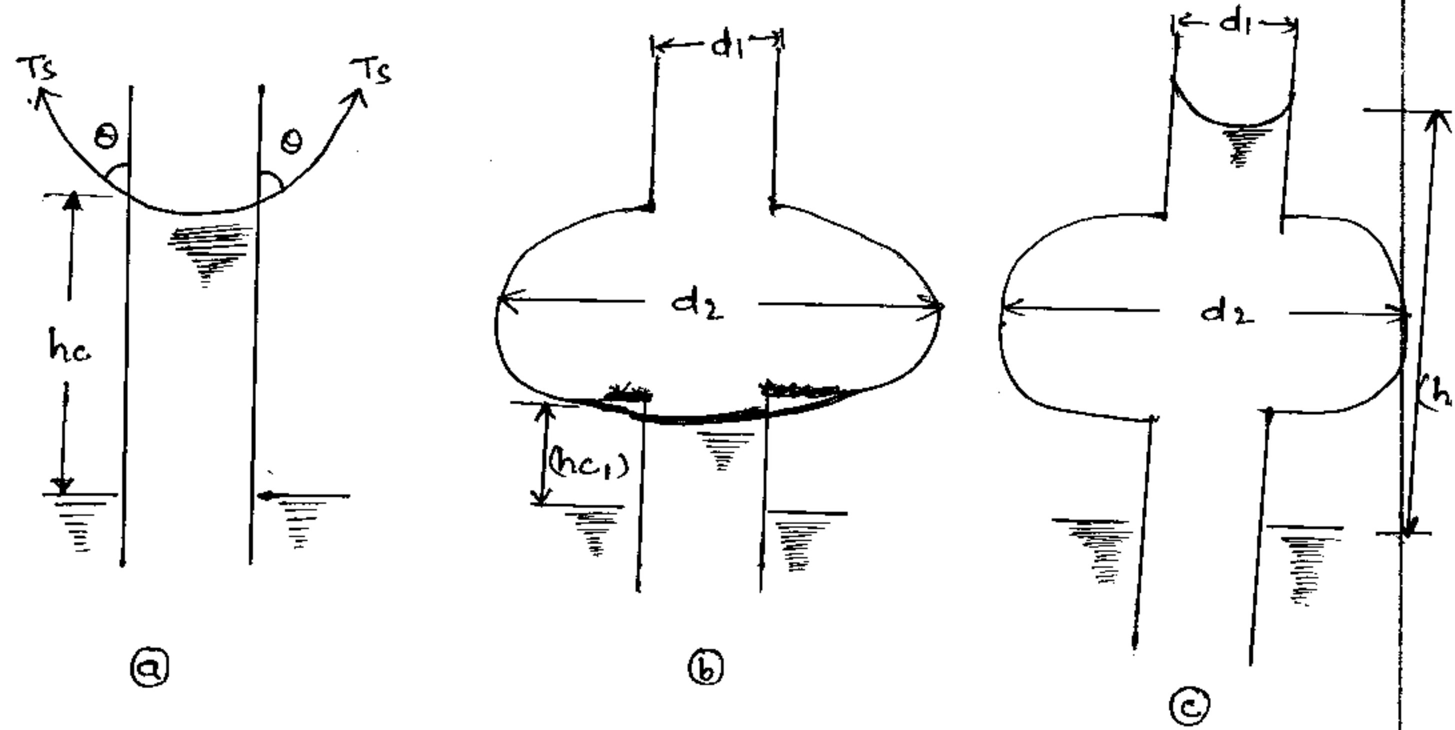
Represen tative hights of capillary Rise.

S.No	Soil type	capillary rise (m)
1.	fine gravel	0.02 to 0.10
2.	Coarse sand	0.10 to 0.15
3.	fin sand	0.30 to 1.00
4.	silt	1.0 to 10.0
5.	clay	10.0 to 30.0
6.	colloid	More than 30.0

Capillary Rise in small diameter tubes :-

Water rises in small diameter, capillary tubes, becouse of adhesion and cohesion. Adhesion occurs becouse water adherse or sticks to the solid walls of the tubes. Cohesion is due to matual atraction of water moleculer. if the effect of cohesion is less significant than the effect of adhesion, the liquid wets the surface and the liquid rises at the point of contact. How ever if the effect of cohesion is more predominant than adhesion, the liquid level is depressed at the point of contact as in the case of mercury.

if a glass tube of small diameter, open at both ends, is lowered in to water, the water level rise in the tube, as the water wets the tube. let 'θ' be the angle of contact between the water and the wall of the tube.



F_u = up ward pull due to surface tension $= (T_s \cos\theta)\pi d$

Where T_s = Surface tension and 'd' diameter of the tube

f_d = Down word force due to mass of water in the tube.

$$= \gamma_w \left(\frac{\pi}{4} d^2\right) \times h_c$$

where h_c = height of capillary rise.

for equilibrium

$$f_u = f_d$$

$$(T_s \cos\theta)\cdot \pi d = \gamma_w \left(\pi/4\; d^2\right) h_c$$

$$h_c = \frac{4 T_s \cos\theta}{\gamma_w d} = \frac{4 T_s \cos\theta}{g \rho_w d}$$

For a clean glass tube and pure water, the meniscus is approximately hemispherical, ie $\theta = 0$

There fore,

$$h_c = \frac{4T_s}{\gamma_w d}$$

Taking $T_s = 0.073\ N/m$, $\gamma_w = 9810\ N/m^3$

$$h_c = \frac{4 \times 0.073}{9810 d} = \frac{3 \times 10^{-5}}{d} \text{ meters.}$$

Where d is in meters.

if d is in centimeters

$$h_c = \frac{3 \times 10^{-3}}{d} \text{ meters.}$$

if h_c and d both are in cm

$$h_c = \frac{0.3}{d} \text{ cm.}$$

Formulas:

1. $h_c = \dfrac{4 T_s \cos\theta}{g \rho_w d}$ (or) $\dfrac{4 T_s \cos\theta}{\gamma_w d}$

2. $h_c = \dfrac{C}{e D_{10}}$

3. $h_c = \dfrac{0.30}{d}$ cm.

→ ① What is the −Ve presure in the water just below the meniscus in a capillary tube of diameter 0.1 mm. filled with water. the surface tension is 0.075 N/m. and wetting angle is 10°.

Sol:

$$h_c = \frac{4 T_s \cos\theta}{\gamma_w d}$$

$$= \frac{4 * 0.075 * 0.9848}{9810 * 0.1 * 10^{-3}} = 0.301 \text{ m.}$$

$$-\text{ve pressure} = \gamma_w h_c = 9.81 * 1000 * 0.301$$

$$= 2952.81 \text{ N/m}^2.$$

→ ②. Estimate the capillary rise in a soil with a void ratio of 0.60 and effective size of 0.01 mm. Take $c = 15 \text{ mm}^2$.

Sol:

$$h_c = \frac{c}{e D_{10}} = \frac{15}{0.6 \times 0.01}$$

$$= 2500 \text{ mm}$$

$$= 2.5 \text{ m.}$$

→③ The capillary rise in a soil A with an effective size of 0.02 mm ; 60 cm Eximate the capillary rise in a similar soil B with an effective size of 0.04 mm.

Sol:-
$$\frac{(h_c)_1}{(h_c)_2} = \frac{(D_{10})_2}{(D_{10})_1}$$

$$\frac{60}{(h_c)_2} = \frac{0.04}{0.02} = 2 \quad (\text{or}) \ (h_c)_2 = 30\,\text{cm}$$

→④. The capillary rise in silt is 50cm and that in fine sand is 30cm. what is the difference in the pore size of the two soils ?

Sol:-
$$h_c = \frac{0.30}{d}\ \text{cm}$$

For silt $(h_c)_1 \Rightarrow 50 = \frac{0.30}{d_1} \Rightarrow d_1 = 6.0 \times 10^{-3}$ cm.

for fine sand $(h_c)_2 \Rightarrow 30 = \frac{0.30}{d_2} \Rightarrow d_2 = 10.0 \times 10^{-3}$ cm.

Difference in pore size $= (10.00 - 6.0) \times 10^{-3}$

$$= 4.00 \times 10^{-3}\ \text{cm}$$

DARCY'S LAW:-

The law of flow of water through soil was first studied by Darcy (1856) who demonstrated expermentaly that for laminar flow conditions in a saturated soil. the rate of flow or the velocity per unit time is proportional to the hydrolic gradient.

$$V \alpha i$$

$$V = ki$$

k = co-eff of permiability

i = hydrolic gradient.

The dicharge 'q' is obtained by multiplying the velocity of flow (V) by the total cross-section area of soil (A) normal to the direction of flow.

Thus $q = VA$

$$q = kiA$$

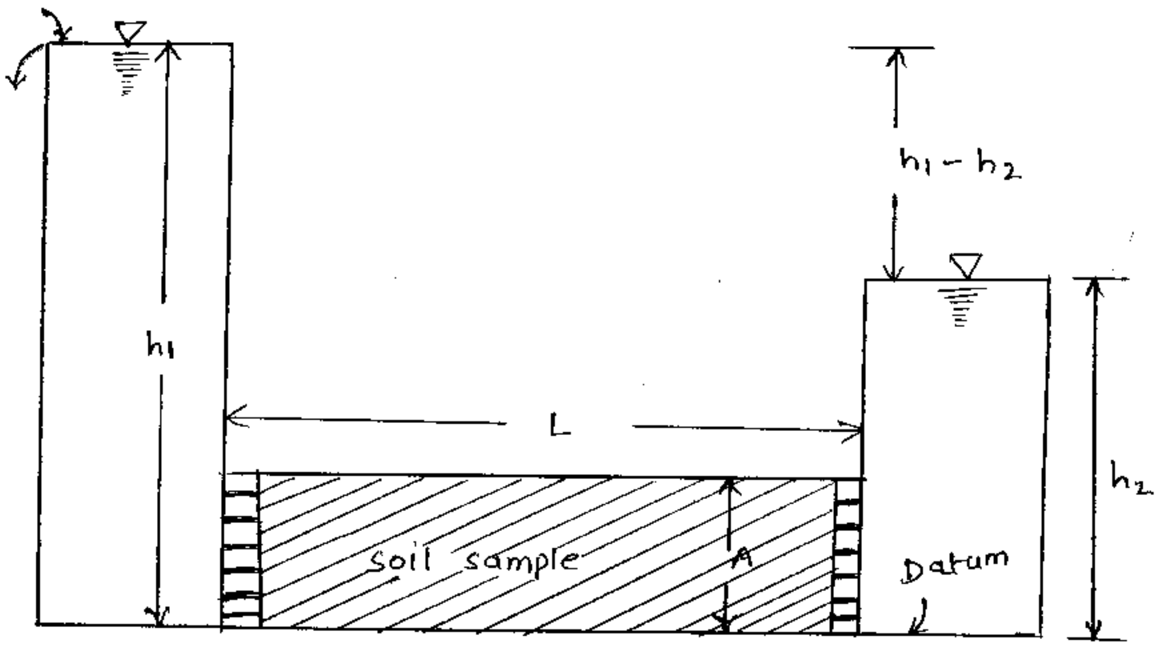
Where, q = discharge per unit time

A = total C/s area of soil mass.

If a soil sample a soil sample of length and C/s area 'A' is subjected to difference head of water $h_1 - h_2$, the hydrolic gradient i will be equal to $\frac{h_1 - h_2}{L}$ and, we have

$$\boxed{q = K \frac{h_1 - h_2}{L} A}$$

Where, i is unity, K is equal to the V. the co-eff of Permiability is defined as Avg velocity of flow that will occur through the total C/s area of soil under unit hydrolic gradient.



* flow of Water Through Soil *

Permeability of soil :-

" The property of a soil Which permits flow of water (or only other liquid) through it is called the permeability of soil" A soil is highly pervious When water can flow through it easily.

In an imperuious soil. the permeability is very low and water can not early flow through it.

Permeability is a very important engineering property of soils. A knowledge of permeability is essential in a number of soil engineering problems. Such as settlement of buildings, yield of wells, seepage through and below the earth structures. it controles the hydrolic stability of soil masses. The permeability of soil is also required in the design of filters used to prevent piping in hydrolic structures.

factors affecting permeability of soil :-

In the laminar flow through porous media.

$$K = c\left(\frac{\gamma_w}{\mu}\right)\left(\frac{e^3}{1+e}\right) D^2$$

Where, -

c = co-efficient

γ_w = unit weight of water

e = void ratio.

μ = viscosity

D = particle size.

The flowing factors affects the permeability of soil.

1. **particle size :-** The co-efficient of permeability of a soil is proportional to the square of the particle size (D). The permeability of coarse-grain soil is very large as compared to that of fine-grained soils. The permeability of coarse sand may be more than one million times as much that of clay.

2. **structure of soil mass :-**

The size of the flow passege depends upon the structural arrangement. For the save void ratio

The permeability is more in the case of floclulated structure as compared to that in the dispersed structure. Stratified soil deposits have greated permeability parallel to the plane of stratification tham that of '⊥' to this plane. permeability of soil ~~deposit also~~ deposite also depends upon shrinkage cracks, joints, fiscures and shear zones. Loss deposits have greater permibility in the vertical direction then in the harizantal direction.

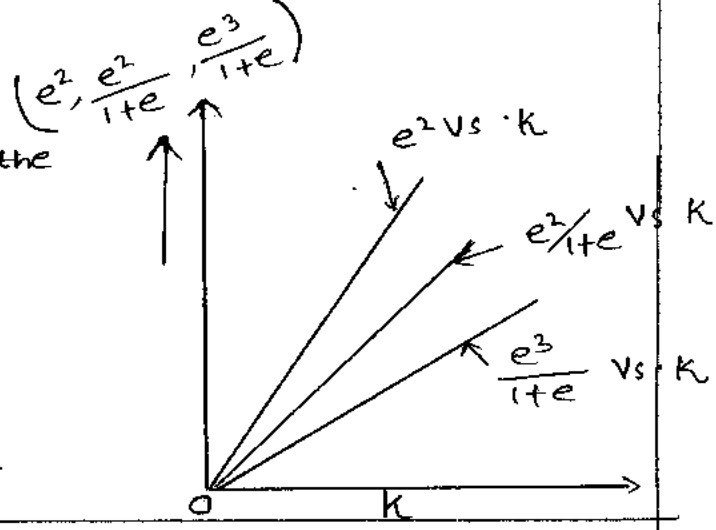
3. Shape of particles :-

The permeability of a soil depends upon the shape of particles. Angular particles have greater specific surface area as compared with the rounded particles. For the same void ratio, the soil with angular particles are less permeable than those with rounded particles.

4. Void ratio :-

The above equation indicated the co-eff of permeability $\frac{e^3}{1+e}$

So the void ratio is greater for the given sample them.



The permiability is also higher the graph plot between the void ratio and co-eff. of permeability is coming a straight line.

→ 5. Property of water :-

The co-efficient of permeability is directly proportional to the unit weight of water (γ_w) and is inversely proportinal to the viscosity (μ). the unit weight of water does not very much over the range of temparature ordinarly encountered in soil engg. problems. How ever there is a large variation in the value of the co-efficient of viscosity (μ). The K increases with an increses in temparature due to reduction in viscosity.

→ 6. Degree of saturation :-

If the soil is not fully saturated, it contains air pockets formed due to entrapped air (or) due to air liberated from percolating water. Whatever may be the cause of the presence of air in soil, the permeability is reduced due to presence of air which causes blockage of passage.

Can sequently, the permeability of a partially saturated soil is smaller than the fully saturated soil.

7. Adsorbed water :-

The fine-grained soil have a layer of adsorbed water strongly attacted to their surface.
this adsorbed water layer is not free to move under gravity. It causes an obstruction to flow of water in the pores and hence reduces the permeability of soil.

8. Impurities in water :-

Any fureign matter in water has a tendency to plug the flow passage and reduce the effective voids and hence the permeability of soils.

Laboratory Methods For permeability test :

The co-efficient of permeability of a soil sample can be determined by the following Methods.

[1] Constant head Method.

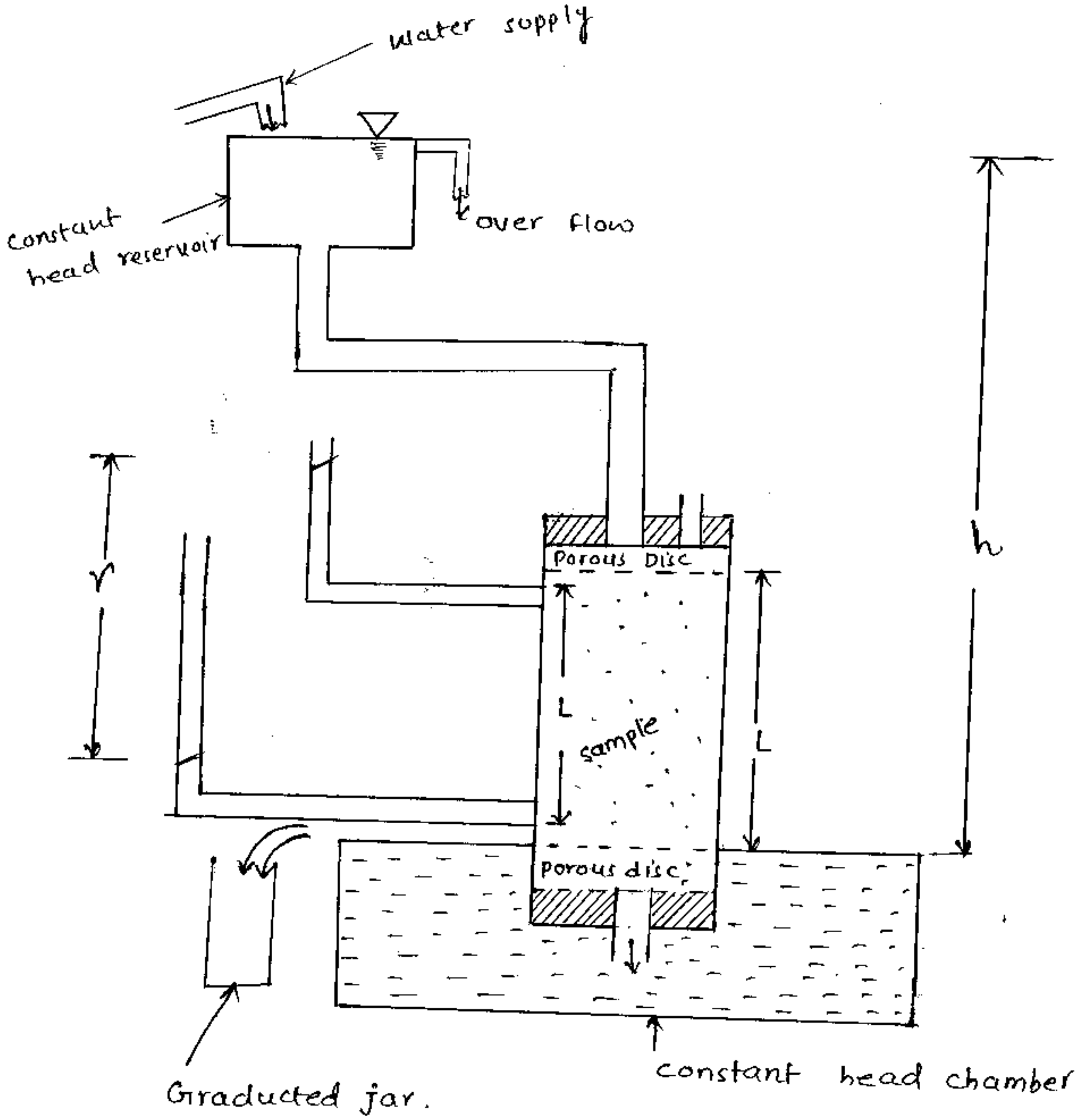
[2]. Variable head Method.

1. Constant head Method :

The co-efficient of permeability of a relatively more permeable soil can be determined in a laboratory by the constant-head permiebility test. The test is conducted in an instrument known as constant-head permeameter test.

It consist of a metalic mould, 100 mm internal diameter, 127.3 mm effective height and 1000 ml capacity according to IS : 2720 (Part XVII). The mould is provided with a detachable extension collar, 100 mm diameter and 60 mm hight, required during compaction of soil. The mould is provided with a drainage base plate with a recess for porous stone. the mould is fitted with a a discharge cap having an inlet valve and an air release valve.

The discharge base and cap have fitting for clamping to the mould.



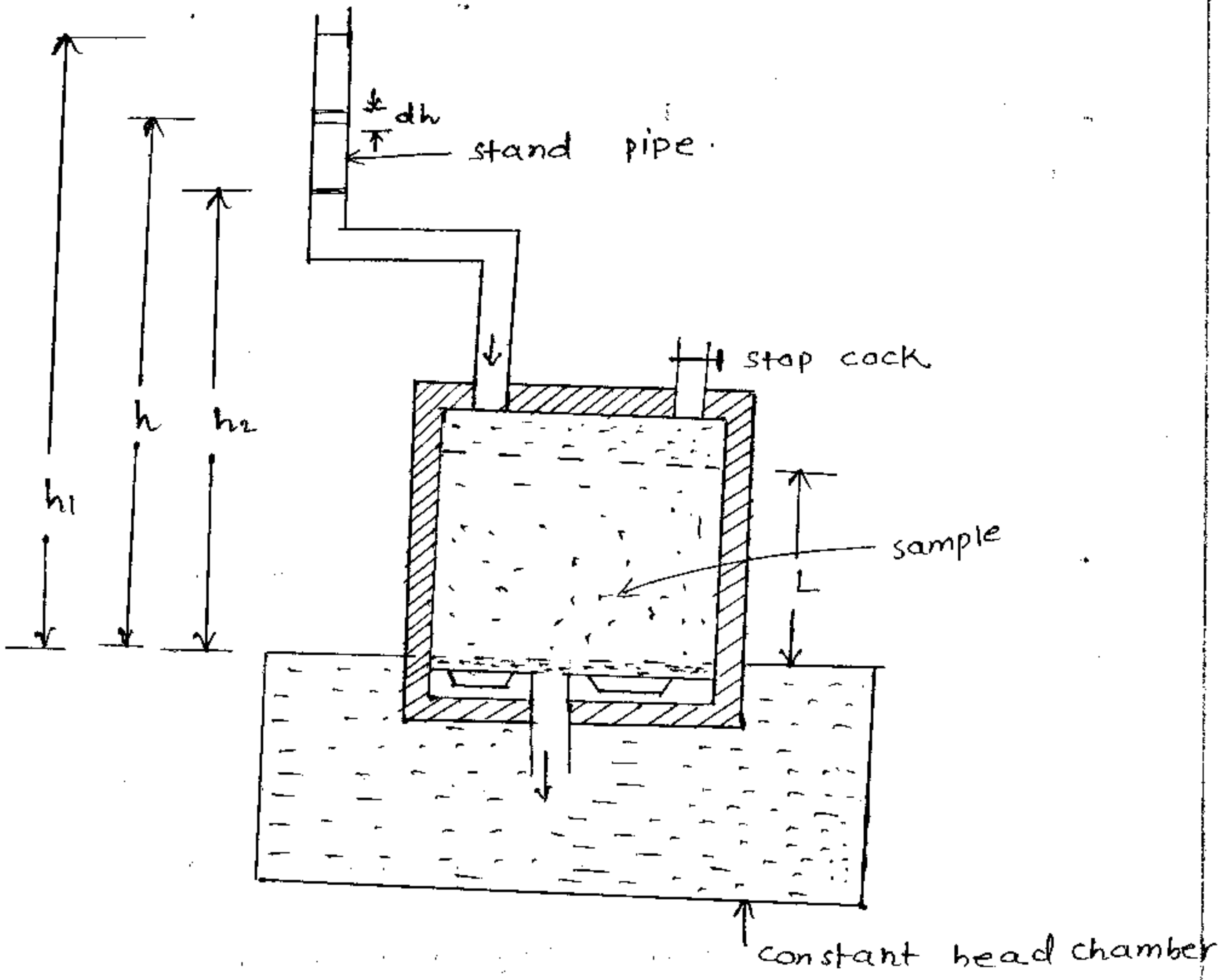
Procedure:

1. Remove the cover of the mould and apply a little grease on the sides of the mould.
2. Measure the internal diameter and effective height of the mould and then attach the collar and the base plate.
3. compact the soil at required dinsity and moisture contant.
4. Remove the collar and base plate. train of ~~&~~ the excess soil level with the top of the mould.
5. Put the porous plate and a filter paper both at top and bottom of the soil sample.
6. place this assemble with washer on the porous stone.
7. Connect the reservoir with water to the inlet at the top of the mould and allow water to flow in till the sample gets saturated.
8. Allow the water to flow through the soil and establish a steady flow.
9. collect the water in a measuring jar for a convenient time interval 't' sec
10. Repeat step (9) for four or five time and tabulat the result as follows.

S.No	Quantity of water collected (V) in c.c	Time of collection t (sec)	$q = V/t$	Head over the sample (H)	$K = \frac{qL}{AH}$ cm/sec
1.					
2.					
3.					
4.					
5					

Variable - Head Permeability test :-

For relatively less permeable soils, the Quantity of water collected in the graduated jar of the constant - head permeability test is very small and can not be measured accurdtely. For such soils, The Variable - head permeability test is used. A verticle, graduated stand pipe of known diameter is fitted to the top of permeameter. the sample is placed between two porous discs. the whole assemble is placed in a constant head chamber Filled with water to the brim at the start of the test.



* Variable head permeameter*

Let us consider the instant when the head is 'h' for the infinitesimal small time 'dt' the head falls by dh. Let the discharge though the sample be q. From continuity of flow.

$$a\,dh = -q\,dt \qquad \boxed{a\,dh = -q\,dt}$$

Where a is cross-sectional area of the stand pipe

$$a\,dh = -(A \times K \times i) \times dt$$

$$a\,dh = -AK \times h/L \times dt$$

$$\frac{AK\,dt}{aL} = -\frac{dh}{h}$$

integrating $\frac{AK}{aL}\int_{t_1}^{t_2} dt = -\int_{h_1}^{h_2} \frac{dh}{h}$

$$\frac{AK}{aL}(t_2 - t_1) = \log_e(h_1/h_2)$$

$$K = \frac{aL}{At}\log_e(h_1/h_2)$$

(or) $$\boxed{K = \frac{2.30\,aL}{At}\log_{10}(h_1/h_2)}$$

Procedure :

Step ① to step ⑥ is same as the constant-head Method procedure.

→ ⑦ connect the stand pipe to the inlet at the top plate and Fill the stand pipe with water.

(8) open the stop clock at the top and allow water to flow out so that all the air in the cylinder is removed

(9) Allow water to flow through the soil and establish a steady flow.

(10) Record the time intervels for the head to fall from h_1 to h_2 for five times, and tebulate the results as follows.

S.NO	h_1	h_2	Time intervel 't'	$K = 2.3 \frac{aL}{At} \log_{10}\left(\frac{h_1}{h_2}\right)$

a = C/s area of stand pipe.

A = C/s area of soil sample

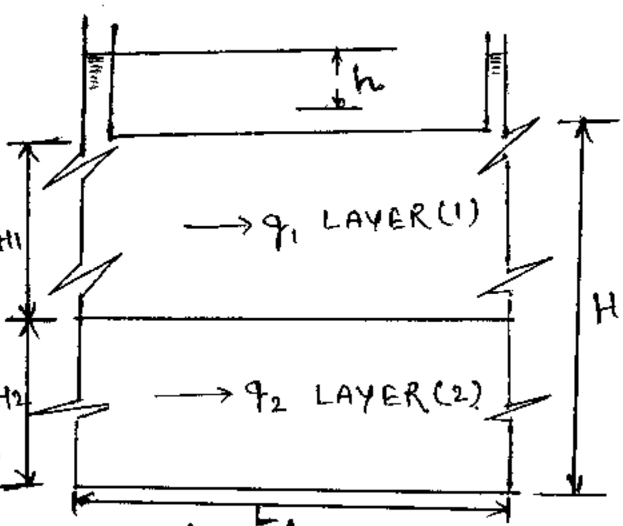
L = length of the soil sample.

Permeability of layered sytems :-

A stratified soil deposit consists of a number of soil layers having different permeabilitys. The Avg permeability of deposit as a whole parallel to the planes of stratification and that normal of the planes of below.

(a) flow parallel to planes of stratification :-

Let us consider a deposit consisting of two horizantal layers of soil of thickness H_1 and H_2 as shown in fig.



for flow parallel to the planes of stratification, the less of head (h) over a length L is the same for the both the layers. there fore, the hydrolic gradient (i) for each layer. is equal to the hydrolic gradient of entir deposite. the system is analogous to the two resistance in parallel in an electrical circuit, where in the potential drop is the same in both the resistance.

From the continuity equation, the total discharge (q) per unit width is equal to the sum of the discharges in the individed layers i.e,

$$q = q_1 + q_2$$

Let $(Kh)_1$ and $(Kh)_2$ be the permeability of the layers 1 and 2 respectively, parallel to the plane of stratifies direction.

from equation (a) using Darcy's law.

$$Kh \times i \times (H_1 + H_2) = (Kh)_1 \times i \times H_1 + (Kh)_2 \times i \times H_2$$

$$Kh = \frac{(Kh)_1 \times H_1 + (Kh)_2 \times H_2}{H_1 + H_2}$$

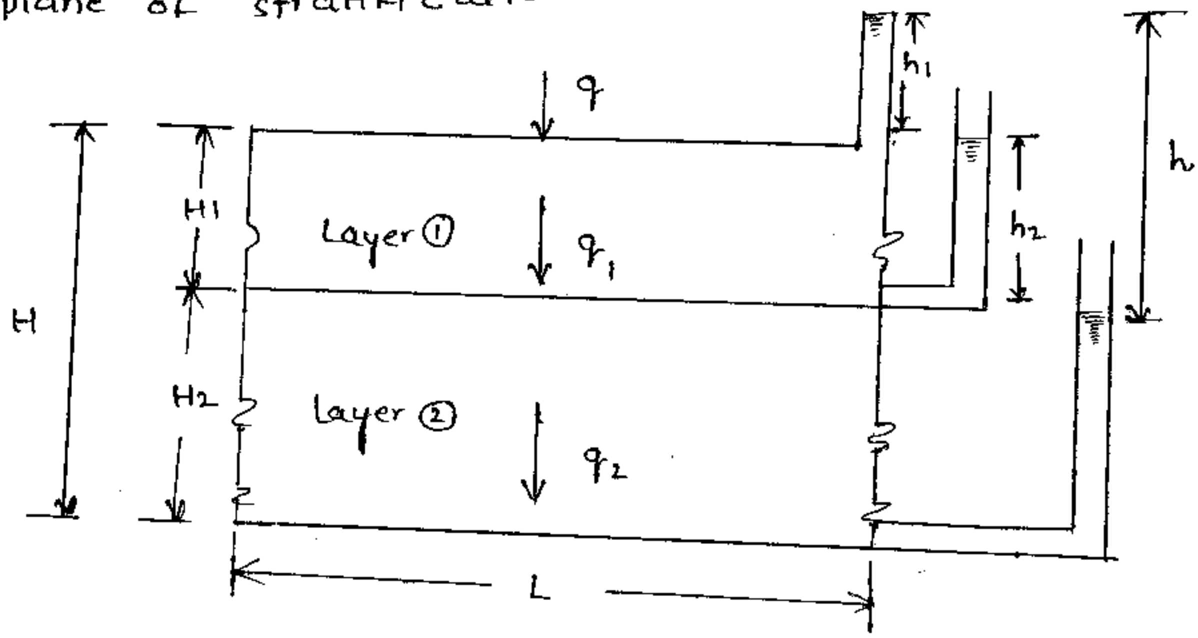
if there are n layers

$$Kn = \frac{(Kn)_1 \times H_1 + (Kn)_2 \times H_2 + \cdots + (Kn)_n \times H_n}{H_1 + H_2 + \cdots + H_n}$$

ⓑ Flow normal to the plane of stratification :-

Let us consider a soil deposit consisting of two layers of thickness H_1 and H_2 in which the flow occurs normal to the

the plane of stratification.



* flow normal to plane of stratification *

Let $(K_V)_1$ and $(K_V)_2$ be the co-efficient of permeability of the layer ① & ② in the direction perpendicular to the plane of stratification and K_V be the average co-eff. of the entire deposit in that direction.

For each layer discharge is equal

$$q = q_1 = q_2 \longrightarrow ①$$

using Darcy's law, considering unit area $\perp$ to flow,

$$K_v \times i_v \times 1 = (K_v)_1 \times (i_v)_1 \times 1 = (K_v)_2 \times (i_v)_2 \times 1 \longrightarrow (2)$$

Where, i_v = overall hydrolic gradient.

$(i_v)_1$ = hydrolic gradient in layer (1)

$(i_v)_2$ = " " " in layer (2).

from equation (2).

$$(i_v)_1 = \left[\frac{K_v}{(K_v)_1}\right] \times i_v \longrightarrow (3)$$

$$(i_v)_2 = \left[\frac{K_v}{(K_v)_2}\right] \times i_v \longrightarrow (4)$$

As the total loss of head (h) over the entire deposite is equal to the sum of the loss of heads in the individual layers

$$h = h_1 + h_2$$

writing in terms of hydrolic gradient (i) and the distance of flow remembering

$$h = i \times e.$$

$$i_v \times H = (i_v)_1 \times H_1 + (i_v)_2 \times H_2$$

using equation 3 & 4

$$i_v \times H = \frac{K_v}{(K_v)_1} \times i_v \times H_1 + \frac{K_v}{(K_v)_2} \times i_v \times H_2$$

$$K_v \left[\frac{H_1}{(K_v)_1} + \frac{H_2}{(K_v)_2} \right] = H = H_1 + H_2$$

$$\boxed{K_v = \frac{H_1 + H_2}{\frac{H_1}{(K_v)_1} + \frac{H_2}{(K_v)_2}}}$$

formulaes :

1. $K = \dfrac{qL}{Ah}$

2. $K = \dfrac{qL}{At} \log_e \left(\dfrac{h_1}{h_2}\right)$

3. $K = c \left(\dfrac{\gamma_w}{\mu}\right) \left(\dfrac{e^3}{1+e}\right) D^2$

4. $K_h = \dfrac{(K_h)_1 \times H_1 + (K_h)_2 \times H_2 + (K_h)_3 \times H_3}{H_1 + H_2 + H_3}$

Problems :-

* In a constant head permeameter test, the following observations were taken

Distance b/w piezometer toppings = 100 mm.

Difference of water levels in piezometers = 60 mm

Diameter of the test sample = 100 mm.

Quantity of the ~~test sample~~ ~~~~ water collected = 350 ml³

Duration of the test = 270 sec.

Determine the co-eff of permeability of the soil.

Sol: We know. $K = \frac{qL}{Ah}$

$$q = V/t = \frac{350}{270} = 1.296 \ ml^3/sec$$

$$K = \frac{1.296 \times 10.0}{\frac{\pi}{4} \times 10^2 \times 6.0} = 0.0275 \ cm/sec$$

*② The falling-head permeability test was conducted on a soil sample of 4 cm diameter and 18 cm length, the head fell from 1.0 m, 0.40m in 20 min. if the c/s area of the stand pipe was 1 cm², determine the co-efficient of permiability.

Sol:

$$K = \frac{qL}{At} \log_e \left(h_1/h_2 \right)$$

$$= \frac{1.0 \times 18.0}{\pi/4 \times 4^2 \times 20 \times 60} \log_e \left(\frac{1.0}{0.40} \right)$$

$$= 1.09 \times 10^{-3} \text{ cm/sec}$$

*③- The co-efficient of permeability of a soil at a void ratio of 0.7 is 4×10^{-4} cm/sec. Estimate its value at a void ratio of 0.50.

Sol:

$$K = c \left(\frac{\gamma_w}{\mu} \right) \left(\frac{e^3}{1+e} \right) D^2$$

As all the parameters remain constent, excepte,

$$\frac{K_{0.7}}{K_{0.5}} = \frac{(0.70)^3}{(0+0.70)} \times \left(\frac{1+0.50}{(0.50)^3}\right)$$

$$= \frac{4 \times 10^{-4}}{K_{0.5}} = 2.421$$

$$K_{0.5} = 1.65 \times 10^{-4} \text{ cm/sec.}$$

UNIT-IV

SEEPAGE THROUGH SOILS

Introduction:-

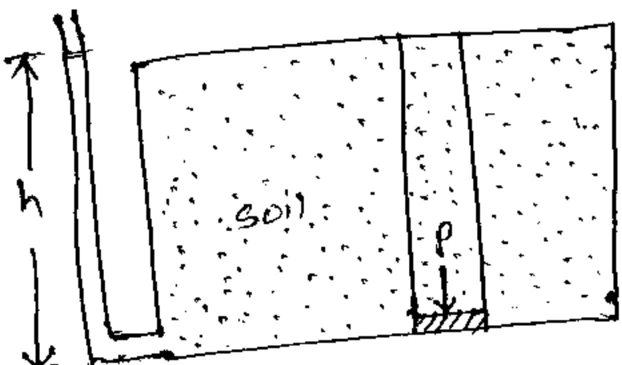
Seepage is the flow of water under gravitational forces in a permeable medium. Flow of water takes place from a point of high head to a point of low head. The flow generally laminar.

The path taken by a water particle is represented by a flow line. Although an infinite number of flow lines can be drawn, for convenience, only a few are drawn. At certain points on different flow lines, the total head will the same. The lines connecting points of equal total head can be drawn. These lines are known as equipotential lines.

Effective stress principle:-

- Definition of effective stress:-

The fig. shows a soil mass which is fully saturated. Let us consider a prism of



Saturated soil mass

soil with a cross-sectional area A. The weight P of the soil in the prism is given by

$$P = \gamma_{sat} h A$$

where γ_{sat} is the saturated weight of the soil, and h is the height of the prism.

Total stress (σ) on the base of the prism is equal to the force per unit area. Thus.

$$\sigma = \frac{P}{A} = \frac{\gamma_{sat} h A}{A}$$

$$\sigma = \gamma_{sat} h \longrightarrow (1)$$

While dealing with stresses, it is more convenient to work in terms of unit weights rater then density.

$$\gamma = \rho g$$

where γ is in N/m^3 and ρ is in kg/m^3, $g = 9.81 m/sec^2$

Thus, $\gamma_{sat} = \rho_{sat} \times g = 9.81 \rho_{sat}$

Generally, the unit weights are expressed in kN/m^3 and the mass density in kg/m^3. In that case,

$$\gamma_{sat} = \frac{\rho_{sat} \times g}{1000} = 9.81 \times 10^{-3} \rho_{sat}$$

For example, if $\rho_{sat} = 2000$ kg/m³

$$\gamma_{sat} = 9.81 \times 10^{-3} \times 2000 = 19.62 \text{ kN/m}^3$$

Pore water pressure (u) is the pressure due to pore water filling the voids of the soil. Thus

$$u = \gamma_w h \longrightarrow (2)$$

Pore water pressure is also known as neutral pressure & neutral stress, because it cannot resist shear stresses.

Pore water pressure is taken as zero when it is equal to atmospheric pressure, because in soil engineering the pressures used are generally gauge pressure and not absolute pressures.

The effective stress ($\bar{\sigma}$) at a point in the soil mass is equal to the total stress minus the porewater pressure. Thus.

$$\bar{\sigma} = \sigma - u \longrightarrow (3)$$

For saturated soils, it is obtained as

$$\bar{\sigma} = \gamma_{sat} h - \gamma_w h$$

$$\bar{\sigma} = (\gamma_{sat} - \gamma_w) h$$

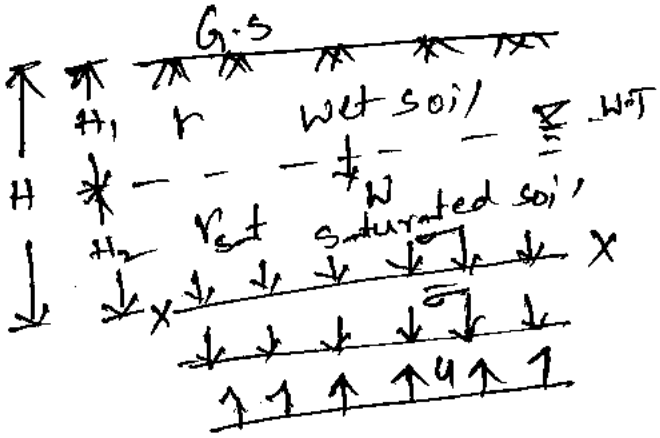
$$\bar{\sigma} = \gamma' h$$

where γ' is the submerged unit weight.

The effive stress is also represented by $\bar{\sigma}'$ in some texts.

Effect of Water Table Fluctuations on Effective Stress:

Let us consider a soil mass shown in the below fig. The depth of the water table (W.T) is H_1 below the ground surface. The soil above the water table is assumed to be wet, with a bulk unit weight of γ. The soil below the water table is saturated, with a saturated weight of γ_w.



Total down ward force (P) at section X-X is equal to the weight (W) of the soil.

Thus

$$P = W = \gamma H_1 A + \gamma_{sat} H_2 A$$

where A is the area of C/S of the soil mass

dividing by A throughout,

$$\frac{P}{A} = \gamma H_1 + \gamma_{sat} H_2$$

The left-hand side is equal to the total stress from eqn (1)

$$\therefore \quad \sigma = \gamma H_1 + \gamma_{sat} H_2$$

The pore water Pressure (u) is given by

$$u = \gamma_w H_2$$

The effect Stress $\bar{\sigma} = \sigma - u$

$$= (\gamma H_1 + \gamma_{sat} H_2) - \gamma_w H_2$$

$$= \gamma H_1 + (\gamma_{sat} - \gamma_w) H_2$$

$$= \gamma H_1 + \gamma' H_2$$

a) If the water table rises to the ground surface and the whole of the soil is saturated, and

$$\bar{\sigma} = \gamma'(H_1 + H_2) = \gamma' H .$$

As $\gamma' < \gamma$, the effective stress is reduced due to rise of water table.

b) If the water table is depressed below the section $x-x$.

$$\bar{\sigma} = \gamma H$$

In this case, the effective stress is increased.

Thus, it is observed that the fluctuations in water table level cause changes in the pore-water pressure and the corresponding changes in the effective Stress.

Effective stress in a soil mass under Hydrostatic conditions:

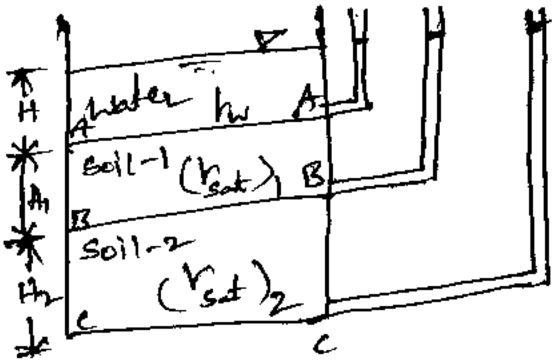


fig (a)

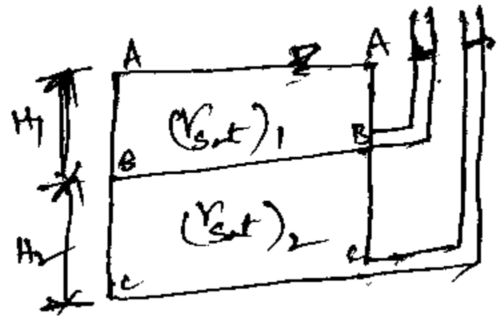


fig (b)

fig(a) shows a soil mass under hydrostatic conditions, where in the water level remains constant. As the interstices in the soil mass are interconnected water rises to the same elevation in different piezometers fixed to the soil mass. The effective stress at various sections can be determined using $\bar{\sigma} = \sigma - u$

1) water table above the soil surface A-A :-

(a) Section A-A :-

$$\sigma = \gamma_w H \ , \ u = \gamma_w H$$

$$\therefore \quad \bar{\sigma} = \sigma - u$$

$$= \gamma_w H - \gamma_w H$$

$$\boxed{\bar{\sigma} = 0} \longrightarrow (1)$$

(b) At section B-B:-

$$\sigma = \gamma_w H + (\gamma_{sat})_1 H_1 \qquad \{\because \bar{\sigma} = \sigma -$$

$$u = \gamma_w (H + H_1)$$

$$\therefore \bar{\sigma} = \left[\gamma_w H + (\gamma_{sat})_1 H_1\right] - \left[\gamma_w (H + H_1)\right]$$

$$= \gamma_w H + (\gamma_{sat})_1 H_1 - \gamma_w H - \gamma_w H_1$$

$$= \left((\gamma_{sat})_1 - \gamma_w\right) H_1$$

$$\boxed{\bar{\sigma} = \gamma_1' H_1} \longrightarrow (2)$$

(c) At section C-C:-

$$\sigma = \gamma_w H + (\gamma_{sat})_1 H_1 + (\gamma_{sat})_2 H_2$$

$$u = \gamma_w (H + H_1 + H_2)$$

$$\bar{\sigma} = \sigma - u$$

$$= \gamma_w H + (\gamma_{sat})_1 H_1 + (\gamma_{sat})_2 H_2 - \gamma_w H - \gamma_w H_1 - \gamma_w H_2$$

$$= \left((\gamma_{sat})_1 - \gamma_w\right) H_1 + \left[(\gamma_{sat})_2 - \gamma_w\right] H_2$$

$$\boxed{\bar{\sigma} = \gamma_1' H_1 + \gamma_2' H_2} \longrightarrow (3)$$

where γ_2' is the submerged unit weight of soil
γ_1' " " " " soil-1

② Water table at the soil surface A-A:-

Fig (b) shows the condition when the depth H of water above the section A-A is reduced to zero. In this case, the effective stresses at various sections are determined as under.

a) At section A-A:-

$$\sigma = 0 \ , \ u = 0$$

$$\bar{\sigma} = \sigma - u$$

$$\boxed{\bar{\sigma} = 0} \longrightarrow (4)$$

(b) At section B-B:-

$$\sigma = (\gamma_{sat})_1 H_1$$

$$u = \gamma_w H_1$$

$$\bar{\sigma} = \sigma - u = (\gamma_{sat})_1 H_1 - \gamma_w H_1$$

$$= [(\gamma_{sat})_1 - \gamma_w] H_1$$

$$\boxed{\bar{\sigma} = \gamma_1' H_1} \longrightarrow (5)$$

(c) At section C-C:-

$$\sigma = (\gamma_{sat})_1 H_1 + (\gamma_{sat})_2 H_2$$

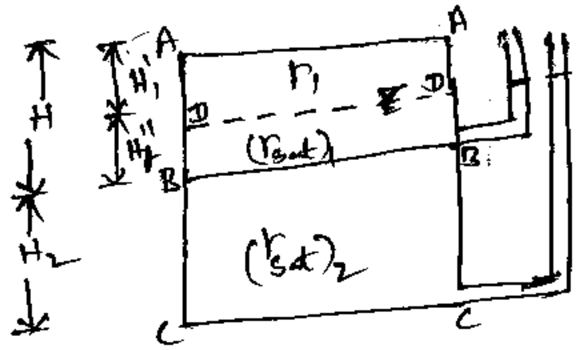
$$u = \gamma_w (H_1 + H_2)$$

$$\bar{\sigma} = \sigma - u = (\gamma_{sat})_1 H_1 + (\gamma_{sat})_2 H_2 - \gamma_w H_1 - \gamma_w H_2$$

$$= [(\gamma_{sat})_1 - \gamma_w] H_1 + [(\gamma_{sat})_2 - \gamma_w] H_2$$

$$\therefore \boxed{\bar{\sigma} = \gamma_1' H_1 + \gamma_2' H_2} \longrightarrow (6)$$

③ Water Table in soil-①:-



The fig show the case when the water table is at D-D in the soil-1 at depth H_1'. The effective stresses at various sections are determined as follows.

ⓐ At section A-A:- $\sigma = 0$, $u = 0$

$$\therefore \bar{\sigma} = \sigma - u = 0$$

$$\boxed{\bar{\sigma} = 0} \longrightarrow (7)$$

ⓑ At section D-D:-

$$\sigma = r_1 H_1', \quad u = 0$$

$$\bar{\sigma} = r_1 H_1' - 0$$

$$\boxed{\bar{\sigma} = r_1 H_1'} \longrightarrow (8)$$

where r_1 is unit weight of soil above D-D.

ⓒ At section B-B:-

$$\sigma = r_1 H_1' + (r_{sat})_1 H_1'' \qquad \{\because H_1' + H_1'' = H_1\}$$

$$u = r_w H_1''$$

$$\therefore \bar{\sigma} = \sigma - u = r_1 H_1' + (r_{sat})_1 H_1'' - r_w H_1''$$

$$= r_1 H_1' + [(r_{sat})_1 - r_w] H_1''$$

$$\boxed{\bar{\sigma} = r_1 H_1' + r_1' H_1''} \longrightarrow (9)$$

(d) At section C-C :-

$$\sigma = \gamma_1 H_1' + (\gamma_{sat})_1 H_1'' + (\gamma_{sat})_2 H_2$$

$$u = \gamma_w H_1'' + \gamma_w H_2'$$

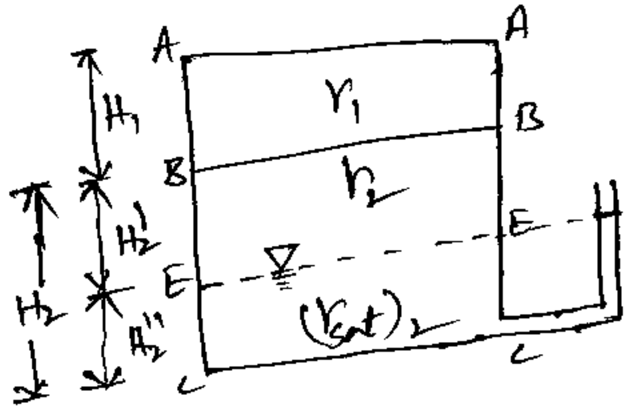
$$\therefore \bar{\sigma} = \sigma - u = \gamma_1 H_1' + (\gamma_{sat})_1 H_1'' + (\gamma_{sat})_2 H_2 - \gamma_w H_1'' - \gamma_w H_2$$

$$= \gamma_1 H_1' + [(\gamma_{sat})_1 - \gamma_w] H_1'' + [(\gamma_{sat})_2 - \gamma_w] H_2$$

$$\boxed{\bar{\sigma} = \gamma_1 H_1' + \gamma_1' H_1'' + \gamma_2' H_2} \longrightarrow (10)$$

(4) Water Table in Soil-2 :-

The fig. shows the condition when the water table is at EE in soil-2 at depth H_2'. The effective stresses at various sections are as under.



(a) Section A-A :- $\sigma = 0$, $u = 0$

$$\therefore \boxed{\bar{\sigma} = 0} \longrightarrow (11)$$

(b) At section B-B :-

$$\sigma = \gamma_1 H_1, \quad u = 0$$

$$\therefore \bar{\sigma} = \sigma - u = \gamma_1 H_1 - 0$$

$$\boxed{\bar{\sigma} = \gamma_1 H_1} \longrightarrow (12)$$

(c) At section E-E :-

$$\sigma = \gamma_1 H_1 + \gamma_2 H_2',$$

$$u = 0$$

$$\bar{\sigma} = \sigma - u = \gamma_1 H_1 + \gamma_2 H_2' - 0$$

$$\therefore \boxed{\bar{\sigma} = \gamma_1 H_1 + \gamma_2 H_2'} \longrightarrow (13)$$

(d) At section C-C :-

$$\sigma = \gamma_1 H_1 + \gamma_2 H_2' + (\gamma_{sat})_2 H_2''$$

$$u = \gamma_w H_2''$$

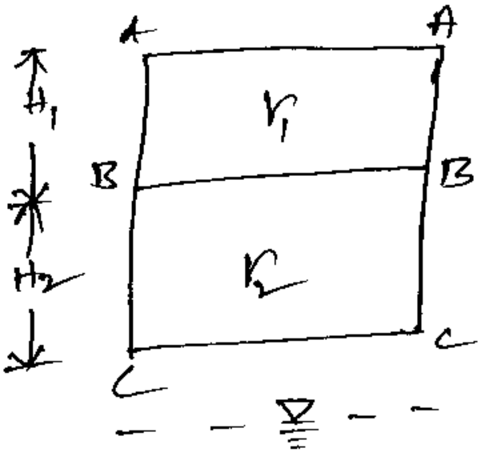
$$\bar{\sigma} = \gamma_1 H_1 + \gamma_2 H_2' + (\gamma_{sat})_2 H_2'' - \gamma_w H_2''$$

$$\bar{\sigma} = \gamma_1 H_1 + \gamma_2 H_2' + \left[(\gamma_{sat})_2 - \gamma_w \right] H_2''$$

$$\therefore \boxed{\bar{\sigma} = \gamma_1 H_1 + \gamma_2 H_2' + \gamma_2' H_2''} \longrightarrow (14)$$

(5) Water Table below C-C :-

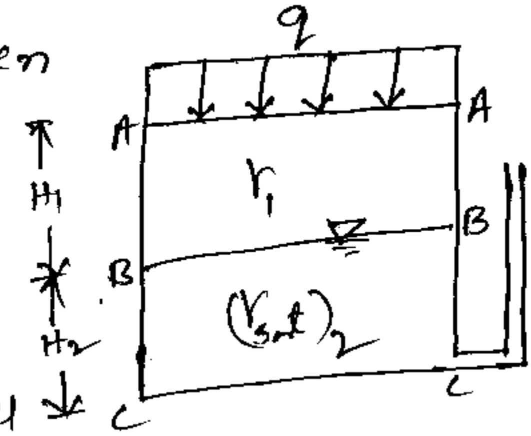
The fig shows the condition when the water table is below C-C. As the pore water pressure is zero everywhere, the effective stresses are also equal to the total stresses.



(a) section B-B :- $\sigma = \bar{\sigma} = \gamma_1 H_1 \longrightarrow (15)$

(b) section C-C :- $\sigma = \bar{\sigma} = \gamma_1 H_1 + \gamma_2 H_2 \longrightarrow (16)$

Increase in Effective Stresses Due To Surcharge :-



Let us consider the case when the soil surface is subjected to a surcharge load of intensity 'q' per unit area. Let us assume that the water table is at level B-B. The stresses at various sections are determined as under.

Section A-A:- $\sigma = q$, $u = 0$

$\therefore \bar{\sigma} = q$

i.e All the points on the soil surface are subjected to an effective stress equal to q.

Section B-B:- $\sigma = q + \gamma_1 H_1$, $u = 0$

$\therefore \bar{\sigma} = q + \gamma_1 H_1$

Section C-C:- $\sigma = q + \gamma_1 H_1 + (\gamma_{sat})_2 H_2$

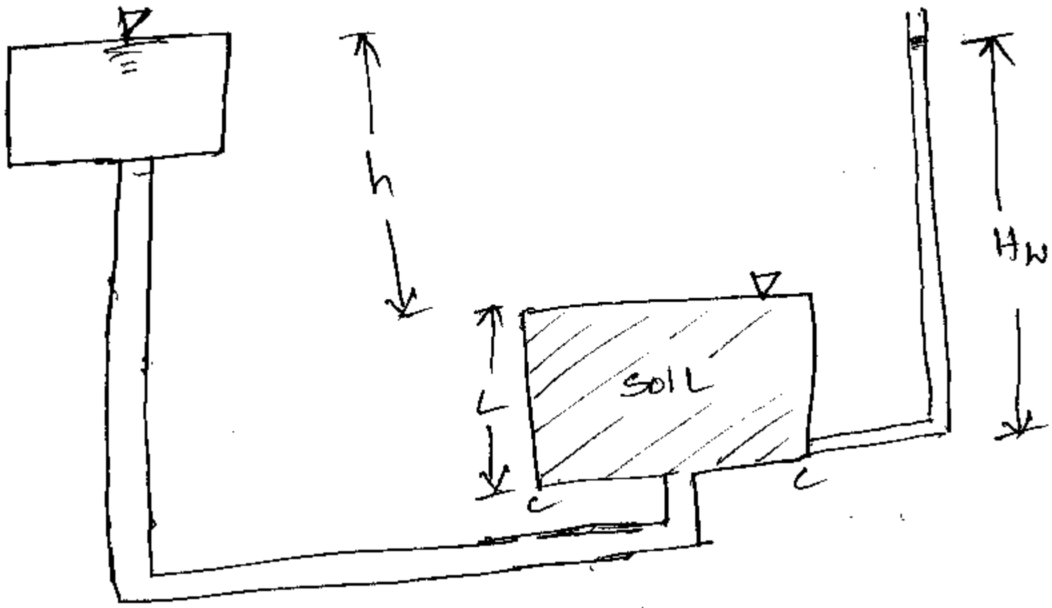
$u = \gamma_w H_2$

$\bar{\sigma} = q + \gamma_1 H_1 + \gamma_2' H_2$

From the above illustrations, it is clear that the effective stress throughout the depth is greater than the case with no surcharge discussed in the preceding section.

Quick Sand Conditions:-

We know the effective stress is reduced due to upward flow of water. When the head causing upward flow is increased, a stage is eventually reached when the effective stress is reduced to zero. The condition so developed is known as quick sand condition.



The above fig. shows a soil specimen of length 'L' subjected to an upward pressure. Let us consider the stresses develop at section c-c.

$$\sigma = \gamma_{sat} L = (\gamma' + \gamma_w) L.$$

$$u = \gamma_w H_{w_1} = \gamma_w (L+h)$$

$$\bar{\sigma} = (\gamma' + \gamma_w) L - \gamma_w (L+h)$$

$$\bar{\sigma} = \gamma' L - \gamma_w h$$

The second term can be written in terms of the hydraulic gradient as under.

$$\gamma_w h = \gamma_w \cdot \left(\frac{h}{L}\right) \times L$$

$$\gamma_w h = \gamma_w i L$$

$$\therefore \quad \bar{\sigma} = \gamma' L - \gamma_w i L$$

The effective stress becomes zero if

$$\gamma' L = \gamma_w i L$$

$$i = \frac{\gamma'}{\gamma_w}$$

~~substituting~~ The hydraulic gradient at which the effective stress becomes zero is known as the critical gradient (i_c). Thus

$$i_c = \frac{\gamma'}{\gamma_w}$$

~~substiting~~ substituting the value of the submerged unit weight in terms of void ratio from the following equation

$$\gamma_{sat} = \frac{(G+e)\gamma_w}{1+e}.$$

$$\gamma' = \gamma_{sat} - \gamma_w.$$

$$\therefore \quad i_c = \frac{\gamma_{sat} - \gamma_w}{\gamma_w} = \frac{\frac{(G+e)\gamma_w}{1+e} - \gamma_w}{\gamma_w}$$

$$= \frac{\frac{(G+e-1-e)\gamma_w}{1+e}}{\gamma_w} \Rightarrow \boxed{i_c = \frac{(G-1)}{1+e}}$$

Taking the specific gravity of solids (G) as 2.67 and the void ratio (e) as 0.67

$$i_c = \frac{2.67-1}{1+1.67} = 1.0$$

Thus the effective stress becomes zero for the soil with above values of 'G' and 'e' when the hydraulic gradient is unity. i.e the head causing flow is equal to the length of the specimen.

Alternative method:-

The above expression for the critical gradient can also be obtained from the equilibrium of forces. when the queck sand condition develops, the upward force is equal to the downward weight.

Thus.
$$\gamma_{sat}(L \times A) = (h+L) A \gamma_w$$

$$(\gamma_{sat} - \gamma_w) LA = Ah\gamma_w$$

$$L\gamma' = h\gamma_w$$

$$\frac{h}{L} = \frac{\gamma'}{\gamma_w}$$

$$\boxed{i = \frac{\gamma'}{\gamma_w}}$$

The shear strength of a cohesionless soil depends upon the effective stress. The shear strength is given by
$$S = \bar{\sigma} \tan\phi$$

The shear strength of cohesive soils is given by

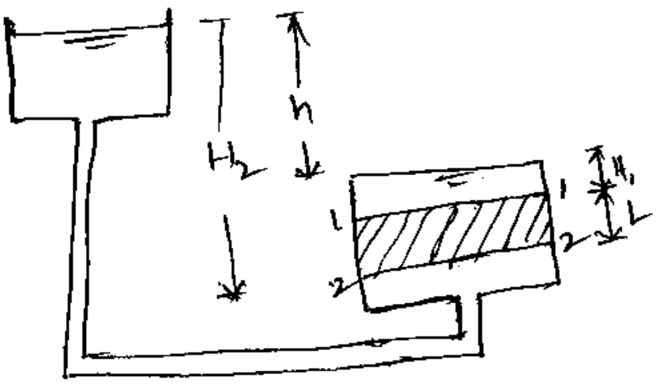
$$S = c + \bar{\sigma} \tan \phi$$

The Quick sand conditions may be summarised as under:

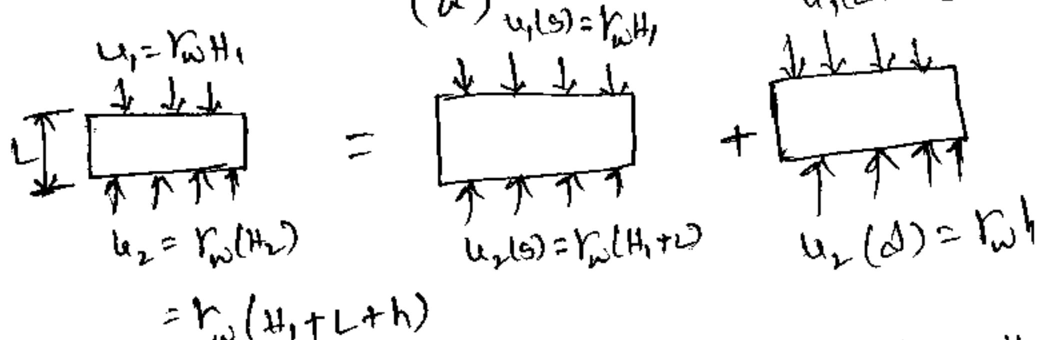
1. Quick sand is not a special type of soil. It is a hydraulic condition.
2. A cohesionless soil becomes quick when the effective stress is equal to zero.
3. The 'i_c' at which a cohesionless soil becomes quick is about unity.
4. The discharge required to maintain a quick condition in a soil increases as the permeability of the soil increases.
5. A quick condition is most likely to occur in silt and fine sand.

Seepage pressure :-

As the water flows through a soil, it exerts a force on the soil. The force acts in the direction of flow in the case of isotropic soils. The force is known as the drag force or seepage force. The pressure induced in the soil is termed seepage pressure.



(a)



(i) Boundary Pressure (ii) Hydrostatic pressure (iii) Hydrodynamic pressure

Fig (b)

Let us consider the upward flow of water in a soil sample of length L and c/s area A under a hydrolic head of h. The expression for seepage force and seepage pressure can be derived considering the boundary water pressure u_1 and u_2 acting on the top and bottom of the soil sample, as shown in fig (b)(i). The boundary water pressure as shown in fig (b) ii, iii can be resolved into two components, namely, the hydrostatic pressure and the hydrodynamic pressure as shown in fig (b) (ii, iii).

1) The hydrostatic pressures $u_1(s)$ and $u_2(s)$ are the components which would occur if there were no flow. If the sample were submerged under water to a depth of H_1, these pressures would have occurred.

2) The hydrodynamic pressure $u_1(d)$ and $u_2(d)$ are the components which are responsible for flow of water. This pressure is spent as the water flows through the soil. These components cause the seepage pressure.

At the top of the sample $u_1 = u_1(s) + u_1(d)$

$$\gamma_w H_1 = \gamma_w H_1 + 0$$

At the bottom of the sample: $u_2 = u_2(s) + u_2(d)$

$$\gamma_w (H_1 + L + h) = \gamma_w (H_1 + L) + \gamma_w h$$

The hydrodynamic pressure is due to hydraulic head. The seepage force (J) acts on the soil skelton due to flowing water through frictional drag. It is given by.

$$J = \gamma_w h A$$

Then seepage pressure (P_s) is the seepage force per unit area, $P_s = \frac{J}{A} = \gamma_w h$.

The seepage pressure (P_s) can be expressed in terms of the hydraulic gradient.

$$P_s = \gamma_w h = \gamma_w (h/L) \cdot L$$

$$P_s = i \gamma_w L$$

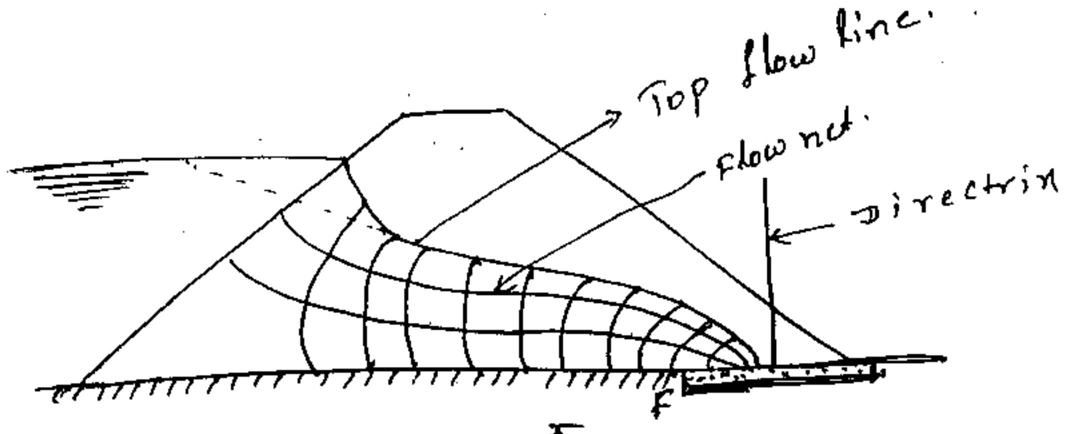
The seepage force (J) can be expressed as the force per unit volume (j), as

$$j = \frac{J}{A \times L} = \frac{\gamma_w h A}{AL} = \gamma_w \frac{h}{L}$$

$$\boxed{j = i \gamma_w}$$

Thus, the seepage force per unit volume is equal to the product of the hydraulic gradient (i) and the unit weight of water.

FLOW NET:-



FLOW NET

Properties of flow net:-

1. The flow lines and equipotential lines meet at right angles to each other.
2. The fields are approximately squares, so that a circle can be drawn touching all the four sides of square.
3. The quantity flowing through each flow channel is the same, similarly, the same potential drop occurs between two succesive equipotential lines.
4. Smaller the dimensions of the field, greater will be the hydralic gradient and velocity of flow through it.
5. In a homogeneous soil, every trasition in the slope of curves is smooth, being either elliptical or Parabolic in slope.

The following Points should be kept in mind while sketching the flow net.

1. Too many flow channels distract the attention from the essential features. Normally three to five flow channels are sufficient. (The space b/n two flow lines is called flow channel)
2. The appearance of the entire flow net ~~has been~~ should be watched and not that of a part of it. small details can be adjusted after the entire flow net has been roughly drawn.
3. The curves should be roughly elliptical (or) parabolic in shape.
4. All transitions should be smooth.
5. The flow lines and equipotential lines should be orthogonal and form approximate squares.
6. The size of the square in a flow channel should change gradually from the upstream to the downstream.

Uses of flow net:-

A flow net can be utilised for the following purpose.

1) Determination of seepage. 2) Determination of hydrostatic pressure
3) Determination of seepage pressure.
4) Determination of exit gradient.

1) Determination of seepage:-

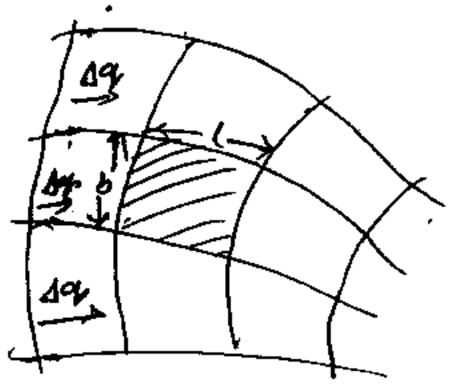


Fig shows a portion of flow net. The portion between any two successive flow lines is known as a flow channel.

The portion enclosed between two successive equipotential lines and successive flow lines is known as 'field' such as that shown hatched.

Let b and l be the width and length of the field.

Δh = head drop through the field; Δq = discharge passing through the flow channel.

H = total hydraulic head causing flow = difference between U/S & D/S heads.

Then, from Darcy's law of flow through soils..

$$\Delta q = k \cdot \frac{\Delta h}{l}(b \times 1) \quad \{\because \text{ considering unit thickness}\}$$

2.) Determination of hydrostatic pressure:-

The hydrostatic pressure at any point within the soil mass is given by $u = h_w \gamma_w$

u = hydrostatic pressure ; h_w = piezometric head.

The hydrostatic pressure in terms of piezometric head h_w is calculated from the following relation.

$$h_w = h - z$$

h = hydraulic potential at point under consider

z = position head of the point above datum, consider positive upward.

All the quantities h_w, h and z can be expressed as the percentage of the total hydralic head H.

eg:- If we want to plot the line of equal pressure corresponding to $h_w = 20\%$ (say)

$h_w = 20 = h - z$ $\quad h_w = 20\%$ on $h = 30\% H$

3) Determination of seepage pressure:-

The hydraulic potential h at any point located after n potential drops, each of value Δh is given by

$$h = H - n\Delta h$$

The seepage pressure at any point equals the hydraulic potential or balance hydraulic head multiplied by the unit weight of water and hence, is given by

$$P_s = h\gamma_w = (H - n\Delta h)\gamma_w$$

The pressure acts in the direction of flow.

4) Determination of exit gradient:- The exit gradient is the hydraulic gradient at the downstream end of the flow line where the percolating water leaves the soil mass and emerges into the free water at the downstream. The exit gradient can be calculated from the following expression, in which Δh represents the potential drop and l the average length of the last field in the flow net at exit end.

$$i_e = \frac{\Delta h}{l}.$$

Problems

1) A coarse-grained soil has a voids ratio of 0.78 and specific gravity as 2.67. calculate the critical gradient at which quick sand condition will occur.

sol)
$$i_c = \frac{\gamma'}{\gamma_w} = \frac{G-1}{1+e}$$
$$= \frac{2.67-1}{1+0.78} = 0.94.$$

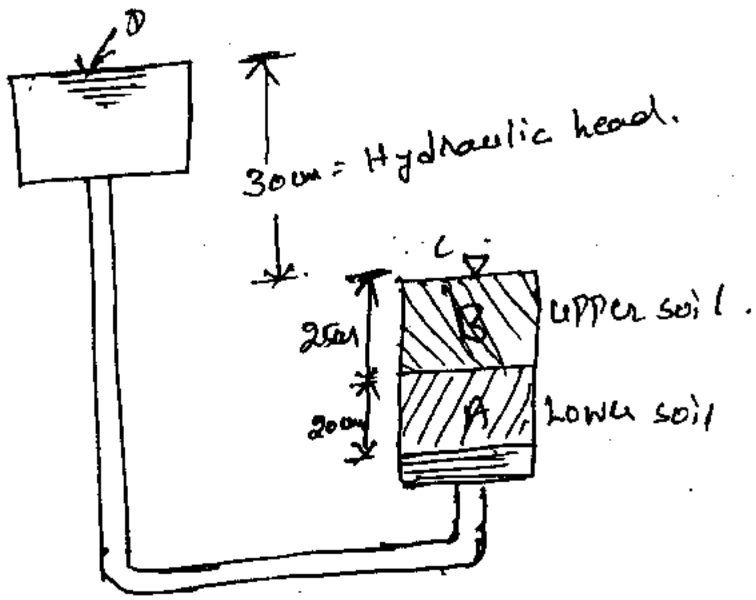
2) In the test set-up shown in figure two different granular soils are placed in permeameter and flow is allowed to takes place under a constant total head of 30 cm.

a) Determine the total head and pressure head at point A.

b) If 30% of the total head is lost as water flows upward through lower soil layer, what is the total head and pressure head at B?

c) If the permeability of layer is 3×10^{-2} cm/sec calculate the quantity of water per second flowing through unit area of the soil.

d) what is the co-eff of permeability of the upper soil level?



sol) Let the water level at C be the datum. The hydraulic head h = 30cm.

a) Total head at D = $h_w + Z$

where h_w = piezometric head (or) pressure head at D = 0

Z = position head ~~at A = h_w + Z~~ at D = 30cm.

∴ Total head at D = 0 + 30 = 30 cm

Total head at A = $h_w + Z$.

h_w = piezometric (or) Pressure head at A.

= 30 + 25 + 20 = 75 cm.

Z = Position head at A = −45 cm.

Total head at A = 75 − 45 = 30 cm = 100% h b). Loss of head from A to B = 30% of of h.

$= \cancel{30}\ 0.3 \times 30 = 9$ cm.

∴ Total head at B = total head at A − head loss in AB

$= 30 - 9 = 21$ cm.

But total head at B = $h_w + z$ when $z = -25$ cm

$21 = h_w - 25$

$h_w = 21 + 25 = 46$ cm = Pr head at I

c) Head lost between A and B = 9 cm.

Now, $q = kiA = k \times \frac{h}{z} \times A$.

Taking $A = 1$ cm^2; $h = 9$ cm, $z = 20$ cm we get

$q = 3 \times 10^{-2} \times \frac{9}{20} \times 1 = 1.35 \times 10^{-2}$ cm^3/sec

d) Same flow takes place through the upper soil

$q = 1.35 \times 10^{-2} = kiA$.

Now, total head at B = 21 cm Total head at c = 0

Head lost between B and C = 21 cm.

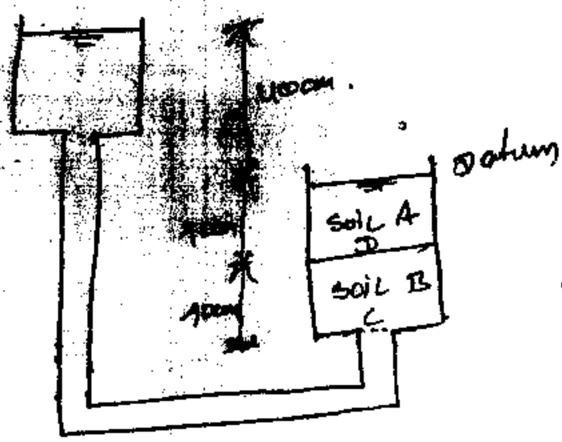
(Alternatively, head loss in upper soil 70% of $h = 0.7 \times (30.0)$)

$\therefore$ i for the upper soil $= 21/25$

$$\therefore \quad k = \frac{q}{iA} = \frac{1.35 \times 10^{-2}}{\frac{21}{25} \times 1}$$

$$= 1.6 \times 10^{-2} \text{ cm/sec}$$

3) In the experimental set up shown in the fig. flow take place under a constant head through the soil A and B



(i) Determine the piezometric head at point c.

(ii) If 40% of the excess hydrostatic pressure is lost in flowing through soil B, what are the hydraulic head and piezometric head at point D.

(iii) If the coefficient of permeability of soil B is 0.05 cm/sec, determine the same for soil A.

(iv) what is the discharge per unit area?

Ans: (i) 120 cm, (ii) 24 cm, 64 cm

(iii) 0.033 cm/sec (iv) 0.02 ml/sec.

UNIT-V

(9)

STRESS DISTRIBUTION IN SOILS

Stress are induced in a soil mass due to weight of overlying soil and due to applied loads. These stresses are required for the stability analysis of the soil mass and in the settlement analysis of the foundation and the determination of earth pressures. The stresses due to self weight of the soil is also called geostatic stresses. The geostatic stresses are two types based on the plane they are acting on. Those are vertical stresses and horizontal stresses.

The vertical stresses are determined by calculating unit weights of the soil layers and porewater pressures in the soil. The stress inducing on soil element at a depth 'Z_0' from the surface of the soil mass is given by

$$\sigma_v = \int_0^{Z_0} \gamma_{Z_0}\, dZ$$

Horizontal stresses are formed by multiplying vertical stresses with some co-efficient. These co-efficients are called co-efficient of earth pressure

The horizontal stress $\sigma_x = K_0 \sigma_v$

Where K_0 is called co-efficient of static earth pressure which is given by

$$\frac{\nu}{1-\nu} \text{ (or) } \frac{1-\sin\phi}{1+\sin\phi}$$

ν = Poisson's ratio

ϕ = Angle of internal friction.

(1) Vertical stresses due to A concentrated load:-

Boussinesq gave the theoretical solutions for the stress distribution in an elastic medium subjected to a concentrated load on its surface.

The solutions are commonly used to obtained the stresses in a soil mass due to externally applied load.

Assumptions of Boussinesq equation :-

1) The soil mass is an elastic continuum, having a constant value of modulus of elasticity (E), i.e, the ratio b/w the stress and strain is constant.

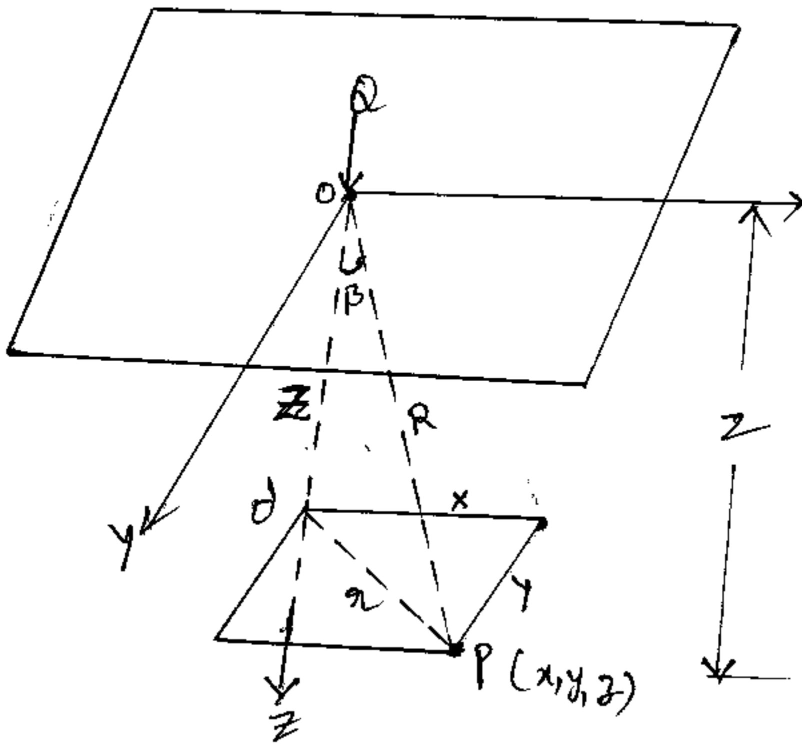
2) The soil is homogeneous, i.e, it has identical properties at different points.

3) The soil is isotropic i.e it has identical properties in all directions.

4) The soil mass is semi-infinite, i.e it extends to infinity in the downwards. In other words, it is limited on its top by horizontal plane and extends to infinity in all other other directions.

(2

5) The soil is weightless and is free from residual stresses before the application of the load.



Stresses due to concentrated load.

The above fig shows a horizontal surface of the elastic continuum subjected to a point load Q at point O. The origin of the co-ordinates is taken at O. Using logarithmic stress function for the solution of elasticity problem. Bossinesq proved that the polar stress σ_R at point P(x, y, z) is given by.

$$\sigma_R = \frac{3}{2\pi} \frac{Q\cos\beta}{R^2}$$

where R = Polar distance between the origin 'O' & point P

β = Angle which the line OP makes with the vertical (Z-axis)

$$r = \sqrt{x^2+y^2} \longrightarrow (1)$$

$$R = \sqrt{r^2+z^2} \longrightarrow (2)$$

$$OP = R = \sqrt{x^2+y^2+z^2} \longrightarrow (3) \quad \{\because r^2 = x^2+y^2\}$$

from $\Delta\, OPO'$

$$\sin\beta = \frac{r}{R} \text{ and } \cos\beta = \frac{z}{R}$$

The vertical stress (σ_z) at the point P is given by

$$\sigma_z = \sigma_R \cos^2\beta$$

$$= \frac{3}{2\pi}\left(\frac{Q\cos\beta}{R^2}\right)\cos^2\beta$$

$$= \frac{3Q}{2\pi}\,\frac{\cos^3\beta}{R^2}$$

$$= \frac{3Q}{2\pi}\,\frac{z^3/R^3}{R^2}$$

$$= \frac{3Q}{2\pi}\,\frac{z^3}{R^5}$$

$$\sigma_z = \frac{3}{2\pi}\,\frac{Q}{z^2} \times \frac{z^5}{R^5} = \frac{3}{2\pi}\,\frac{Q}{z^2}\left(\frac{z}{R}\right)^5$$

Now sub R value from (2)

$$\sigma_z = \frac{3}{2\pi}\,\frac{Q}{z^2}\left(\frac{z}{\sqrt{r^2+z^2}}\right)^5$$

$(r^2+z^2)^{5/2}$

$z^{2 \cdot 5/2}$

$\frac{r}{z}$

$$= \frac{3}{2\pi}\,\frac{Q}{z^2}\,\frac{z^5}{z^5\left(1+\left(\frac{r}{z}\right)^2\right)^{5/2}}$$

$$\sigma_z = \frac{3}{2\pi}\,\frac{Q}{z^2}\left[\frac{1}{\left[1+\left(\frac{r}{z}\right)^2\right]^{5/2}}\right]$$

$$\sigma_z = I_B \times \frac{Q}{z^2}$$

$$\text{where, } I_B = \frac{3}{2\pi\left(1+\left(\frac{r}{z}\right)^2\right)^{5/2}}$$

where I_B is known as Boussinesq's influence co-efficient for verticle stress.

The vertical stress exactly below the load than $r=0$

$$I_B = \frac{3}{2\pi} = 0.4775$$

$$\therefore \sigma_z = 0.4775 \frac{Q}{z^2}$$

This obtained by substituting $r=0$ and $z=z$ in the expression.

Observations :-

The following Points are worth noting when using the above expression.

1) The verticle stress does not depending upon the modul of elasticity (E) and poisson's ratio (μ). But the solut has been derived assuming that the soil is linearly elastic. That means the stresses that forms with in the region of elasticity are lesser than the shear strength of the soil.

2) The intensity of vertical stress just below the load point is given by

$$\sigma_z = 0.4775 \frac{Q}{z^2}$$

3) At the surface (z=0). The vertical stress just below the load is theoritically infinite. However in an actual case the soil under the load ~~a~~ yields due to very high stresses. The load point spreads over a small but finite area. There fore only finite stresses will develope.

4) The vertical stress (σ_z) decreases rapidly with an increase in r/z ratio.

5) Boussinesq's solution can be used for negative loads.

limitations of Boussinesq's soloution

(1) The solution was initially obtained for determination of stresses in elastic solids application to soils may be questioned, as the soils are far from purely elastic solids. However experience indicates that the results obtained are ~~st~~ satisfactoory.

(2) The application of Boussinesq's solution can be justified when the stress changes are such that only a stress increase occurs in the soil.

(3) when the stress decrease occurs, the relation between stress and strain in not linear and, therefore, the solution is not strictly applicable

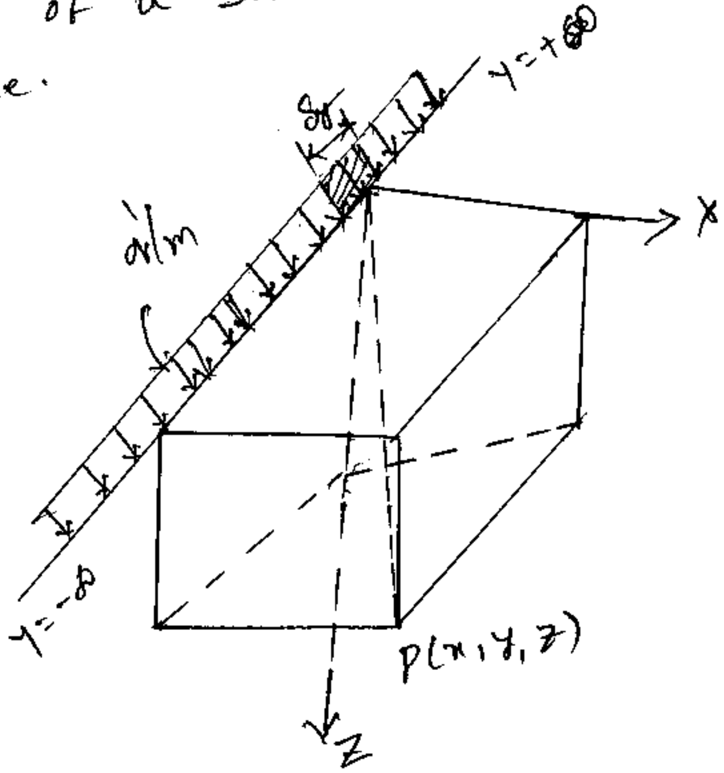
(4) For practical cases, the Boussinesq, solution can be safely used for homogeneous deposits of clay man-made fill and for limited thickness of uniform sand deposits.

(5) The point loads applied below ground surface cause somewhat smaller stresses than are caused by surface loads, and, therefore, the Boussinesq solution is not strictly applicable. However, the solution is frequently used for shallow footings in which z is measured below the of the footing.

(ii) Vertical stress under a line load :-

The expression for vertical stress at any point 'p' under a line load can be obtained by integrating the expression the vertical stress for a point load along the line of load.

Let the vertical line load be of intensity q' for unit length along the y-axis, acting on the surface of a semi infinite soil mass as shown in the figure.



Let us considered the load acting on a small length δy. The load can be taken as a point load of q' δy and boussinesq's solution can be applied to determine the vertical stress at point P(x, y, z) so the normal stresses due to this point load

$$\Delta\sigma_z = \frac{3q'dy}{2\pi} \frac{z^3}{(r^2+z^2)^{5/2}}$$

The vertical stress at P due to the line load extending from $-\infty$ to $+\infty$ is obtained by Integrating

$$\therefore \sigma_z = \frac{3z^3}{2\pi} \int_{-\infty}^{+\infty} \frac{q'}{(r^2+z^2)^{5/2}} dy$$

$$= \frac{3z^3}{2\pi} \int_{-\infty}^{+\infty} \frac{q'}{(x^2+y^2+z^2)^{5/2}} dy$$

substitute $x^2+z^2 = u^2$

$$\sigma_z = \frac{3z^3}{2\pi} \int_{-\infty}^{+\infty} \frac{q'}{(u^2+y^2)^{5/2}} dy$$

$$= \frac{3q'z^3}{2\pi} \int_{-\infty}^{+\infty} \frac{dy}{(u^2+y^2)^{5/2}}$$

Take $y = u\tan\theta$ $\quad \theta \to \frac{-\pi}{2}$ to $\frac{\pi}{2}$

$dy = u\sec^2\theta \, d\theta$

$$\therefore = \frac{3q'z^3}{2\pi} \int_{-\pi/2}^{\pi/2} \frac{u\sec^2\theta \, d\theta}{(u^2+u^2\tan^2\theta)^{5/2}}$$

$$= \frac{3q'z^3}{2\pi} \int_{-\pi/2}^{\pi/2} \frac{u\sec^2\theta \, d\theta}{u^{5} (1+\tan^2\theta)^{5/2}}$$

$$= \frac{3q' z^3}{2\pi} \int_{-\pi/2}^{\pi/2} \frac{u \sec^2\theta \, d\theta}{u^5 \sec^5\theta}$$

$$= \frac{3q' z^3}{2\pi} \int_{-\pi/2}^{\pi/2} \frac{d\theta}{u^4 \sec^3\theta}.$$

$$= \frac{3q' z^3}{2\pi u^4} \int_{-\pi/2}^{\pi/2} \cos^3\theta \, d\theta$$

$$= \frac{3q' z^3}{2\pi u^4} \int_{-\pi/2}^{\pi/2} \cos\theta \, (1-\sin^2\theta) \, d\theta.$$

$t = \sin\theta.$
$dt = \cos\theta \, d\theta.$
$t \to -1 \text{ to } 1$

$$= \frac{3q' z^3}{2\pi u^4} \int_{-1}^{1} 1-t^2 \, dt.$$

$$= \frac{3q' z^3}{2\pi u^4} \left(t - \frac{t^3}{3} \right)_{-1}^{1}$$

$\left(1-\frac{1}{3}-\left(-1-\frac{-1}{3}\right)\right)$
$\left(1-\frac{1}{3}+1-\frac{1}{3}\right)$
$2\left(1-\frac{1}{3}\right)$
$2\left(\frac{2}{3}\right)$

$$= \frac{3q' z^3}{2\pi u^4} \, 2\left(1-\frac{1}{3}\right)$$

$$= \frac{3q' z^3}{\pi u^4} \left(\frac{2}{3}\right) = \frac{2q' z^3}{\pi u^4} = \frac{2q' z^3}{\pi (x^2+z^2)^2}$$

$$= \frac{2q' z^3}{\pi z^4 \left(1+\left(\frac{x}{z}\right)^2\right)^2}$$

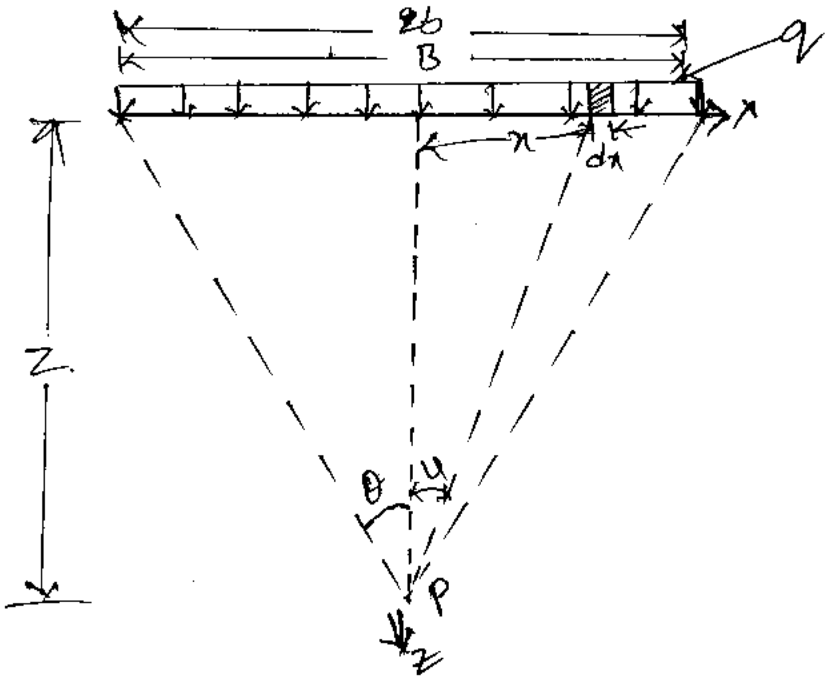
$$\boxed{\sigma_z = \frac{2q'}{\pi z \left(1+\left(\frac{x}{z}\right)^2\right)^2}}$$

(iii) vertical Stress under a strip load:-

The expression for verticle stress at any point P under a strip load can be develope from the expression developed for line load. The expression will depend upon wheather the point P lies below the centre of the strip load or not.

Note:- The length of the strip is very long. For convenienc unit length is considered.

Point is below the centre of the strip. The following figur shows a strip load of with B(=2b)



Take an element of width dx from the distance x from the centre point of the strip load. The small load of qdx can be considered as a line load of intensity q'.

The verticle stress acting at the point P.

$$\Delta\sigma_z = \frac{2q\,dx}{\pi z(1+(x/z)^2)^2}$$

$$= \frac{2q\,dx}{\pi z}\left[\frac{1}{1+(\frac{x}{z})^2}\right]^2$$

The stress due to entire strip load is obtained by integration

$$\sigma_z = \frac{2q}{\pi z}\int_{-b}^{b}\left[\frac{1}{1+(\frac{x}{z})^2}\right]^2 dx$$

substitute $\frac{x}{z} = \tan u$ than $\frac{dx}{dz} = \sec^2 u\,du$

$dx = z\sec^2 u\,du$.

Take $\theta = \tan^{-1}(\frac{b}{z})$ then

$x = b \quad u = \tan^{-1}(\frac{b}{z}) = \theta$

$x = -b \quad u = -\tan^{-1}(\frac{b}{z}) = -\theta$

$$\sigma_z = \frac{2q}{\pi z}\int_{-\theta}^{\theta}\frac{z\sec^2 u}{(1+\tan^2 u)^2}\,du$$

$\frac{1}{\sec\theta} = \cos\theta$

$$= \frac{2q}{\pi z}\cdot 2\int_{0}^{\theta}\frac{z\sec^2 u}{\sec^4 u}\,du$$

$$= \frac{4q}{\pi z}\int_{0}^{\theta} z\cos^2 u\,du = \frac{4q}{\pi}\int_{0}^{\theta}\cos^2 u\,du$$

$$= \frac{4q}{\pi}\int_{0}^{\theta}\left(\frac{1+\cos 2u}{2}\right)du = \frac{2q}{\pi}\left(u+\frac{\sin 2u}{2}\right)_0^{\theta}$$

$$= \frac{2q}{\pi}\left(\theta+\frac{\sin 2\theta}{2}\right)$$

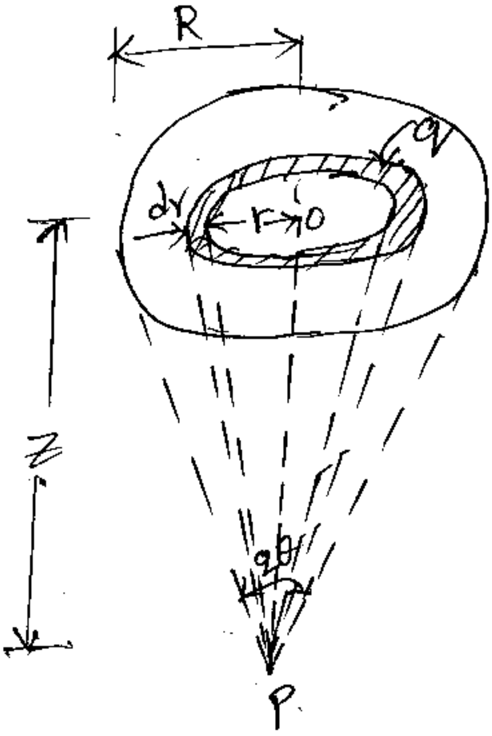
$$\boxed{\sigma_z = \frac{q}{\pi}(2\theta+\sin 2\theta)}$$

Vertical Stress under a circular Area :-

The load applied to soil surface by footings are not ~~quanted~~ concentrated loads. These are usually spread over a finite area of footing. It is generally assumed that the footing is flexiable and the contact pressure is uniform. In other words the load is assumed to be uniformly distributed over the area of base of footings.

Let us determine the vertical stress at the point P at depth 'z' below the centre of a uniformly loaded circular area. Let the intensity of the load be 'q' per unit area, and 'R' be the radious of the loaded area.

The load on the elementary ring of radius 'R' and width 'dr' is equal to $q 2\pi r dr$. The load acts at a constant radia distance 'r' form the point P'. This load acts as a point load. The vertical stress due to this point load



$$\Delta\sigma_z = \frac{3q\,2\pi r dr}{2\pi z^2}\,\frac{1}{\left[1+\left(\frac{r}{z}\right)^2\right]^{5/2}}$$

$$= \frac{3q}{z^2\left[1+\left(\frac{r}{z}\right)^2\right]^{5/2}} \, r\,dr$$

$$\Delta\sigma_z = \frac{3q\,z^3}{(r^2+z^2)^{5/2}} \, r\,dr$$

The vertical stress due to entaire load is given by integrating the above expression with in the limit 0 to R

$$\sigma_z = \int_0^R \frac{3q\,z^3}{(r^2+z^2)^{5/2}} \, r\,dr$$

$$= 3qz^3 \int_0^R \frac{r}{(r^2+z^2)^{5/2}} \, dr$$

substitute $r^2+z^2=u$

$2r\,dr = du$

$r\,dr = \frac{1}{2}du$

$r=0 \Rightarrow u = z^2$

$r=R \Rightarrow u = R^2+z^2$

$$\sigma_z = 3qz^3 \int_{z^2}^{R^2+z^2} \frac{\frac{1}{2}du}{u^{5/2}}$$

$$= \frac{3}{2}qz^3 \int_{z^2}^{R^2+z^2} u^{-5/2}\,du = \frac{3}{2}qz^3 \left[\frac{u^{-5/2+1}}{-\frac{5}{2}+1}\right]_{z^2}^{R^2+z^2}$$

$$= \frac{3}{2} q z^3 \times \frac{-2}{3} \left(u^{-3/2}\right]_{z^2}^{R^2+z^2}$$

$$= -q z^3 \left[(R^2+z^2)^{-3/2} - (z^2)^{-3/2}\right]$$

$$\sigma_z = -q z^3 (R^2+z^2)^{-3/2} + q z^3 z^{-3}$$

$$= q\left[1 - z^3 (R^2+z^2)^{-3/2}\right]$$

$$= q\left[1 - z^3 \cdot z^{-3} \left(1+\left(\tfrac{R}{z}\right)^2\right)^{-3/2}\right]$$

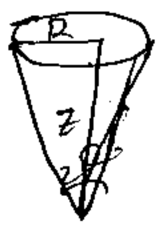
$$= q\left[1 - \left(1+\left(\tfrac{R}{z}\right)^2\right)^{-3/2}\right]$$

$$\sigma_z = q\left[1 - \frac{1}{\left(1+(R/z)^2\right)^{3/2}}\right]$$

$$\sigma_z = I_c q$$

where $I_c = \left[1 - \frac{1}{\left(1+(R/z)^2\right)^{3/2}}\right]$

Value of I_c in terms of θ:-



from fig

$$\tan\theta = \frac{R}{z}$$

$$I_c = 1 - \frac{1}{\left(1+\left(\frac{R}{z}\right)^2\right)^{3/2}}$$

$$= 1 - \frac{1}{(1+\tan^2\theta)^{3/2}}$$

$$= 1 - \frac{1}{(\sec^2\theta)^{3/2}}$$

$$\boxed{I_c = 1 - \frac{1}{\sec^3\theta} = 1 - \cos^3\theta}$$

Vertical stress under a corner of Rectangular Area:-

The vertical stress under a corner of rectangular area with a uniformly distributed load of intensity q can be obtained from Boussinesq's solution. From stress due to point load equation the stress at depth z is given by, taking $dQ = q\,dA = q\,dx\,dy$

$$\Delta\sigma_z = \frac{3(q\,dx\,dy)z^3}{2\pi}\frac{1}{(x^2+y^2+z^2)^{5/2}}$$

By integration

$$\sigma_z = \frac{3qz^3}{2\pi}\int_0^L\int_0^B \frac{q\,dx\,dy}{(x^2+y^2+z^2)^{5/2}}$$

Although the integral is quite complicated. Newmark was able to perform it the result were presented as follows.

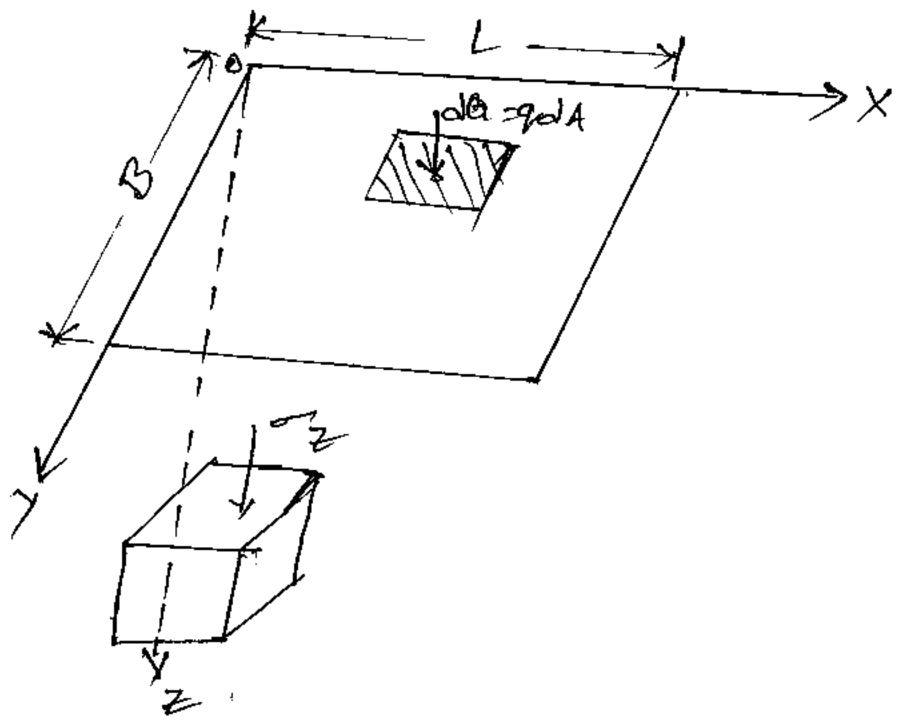
$$\sigma_z = \frac{q}{2\pi}\left[\frac{mn}{\sqrt{m^2+n^2+1}}\,\frac{m^2+n^2+2}{m^2+n^2+m^2n^2+1} + \sin^{-1}\left(\frac{mn}{\sqrt{m^2+n^2+m^2n^2+1}}\right)\right]$$

where $m = \frac{B}{z}$ and

$n = \frac{L}{z}$

The values of m and n can be interchanged without any effect on the values of σ_z.

$$\therefore \boxed{\sigma_z = I_N q}$$



wher I_N is Newmark's influence co-efficient, given by

$$I_n = \frac{1}{2\pi}\left[\frac{mn}{\sqrt{m^2+n^2+1}}\cdot\frac{m^2+n^2+2}{m^2+n^2+m^2n^2+1} + \sin^{-1}\left(\frac{mn}{m^2+n^2+m^2n^2}\right.\right.$$

Newmark's Charts :-

Newmarks influence charts are prepared based on boussinesq's solution to find the vertical stresses generated by different types of footings. These vertical stresses due to circular loads are taken as basis for preparing the charts.

In these charts the circles whoes area is in proportion to the vertical stresses which are generated by the footings which are in same size as though circles in the charts.

In practice some times the engineer as to find the vertical stresses under a uniformly loaded area as of other shapes. In such cases. Newmark influence in compressible, these persists ~~only down ward displacement of the soil mass~~ charts are extremely used. Newmark's chart in based on the concept of the vertical stress below the centre of circular area.

Let us consider a uniformly loaded circular area of radius R_1 divided into 20 equal sectors.

The vertical stress at point 'p' at depth 'z' just below the centre of loaded area due to load on one sector will be $\frac{1}{20}$th of that due to load on full circle.

$$\sigma_z = \frac{1}{20} q \left[1 - \left(\frac{1}{1 + (R/z)^2} \right)^{3/2} \right]$$

In the vertical stress σ_z is given an orbitary fixed value say $0.005q$

sub. above value in expression

$$\frac{R_1}{z} = 0.27$$

That means every $1/20$th sector of the circle with a radius R_1 equal to $0.27z$ would give a vertical stress of $0.005q$ at its centre.

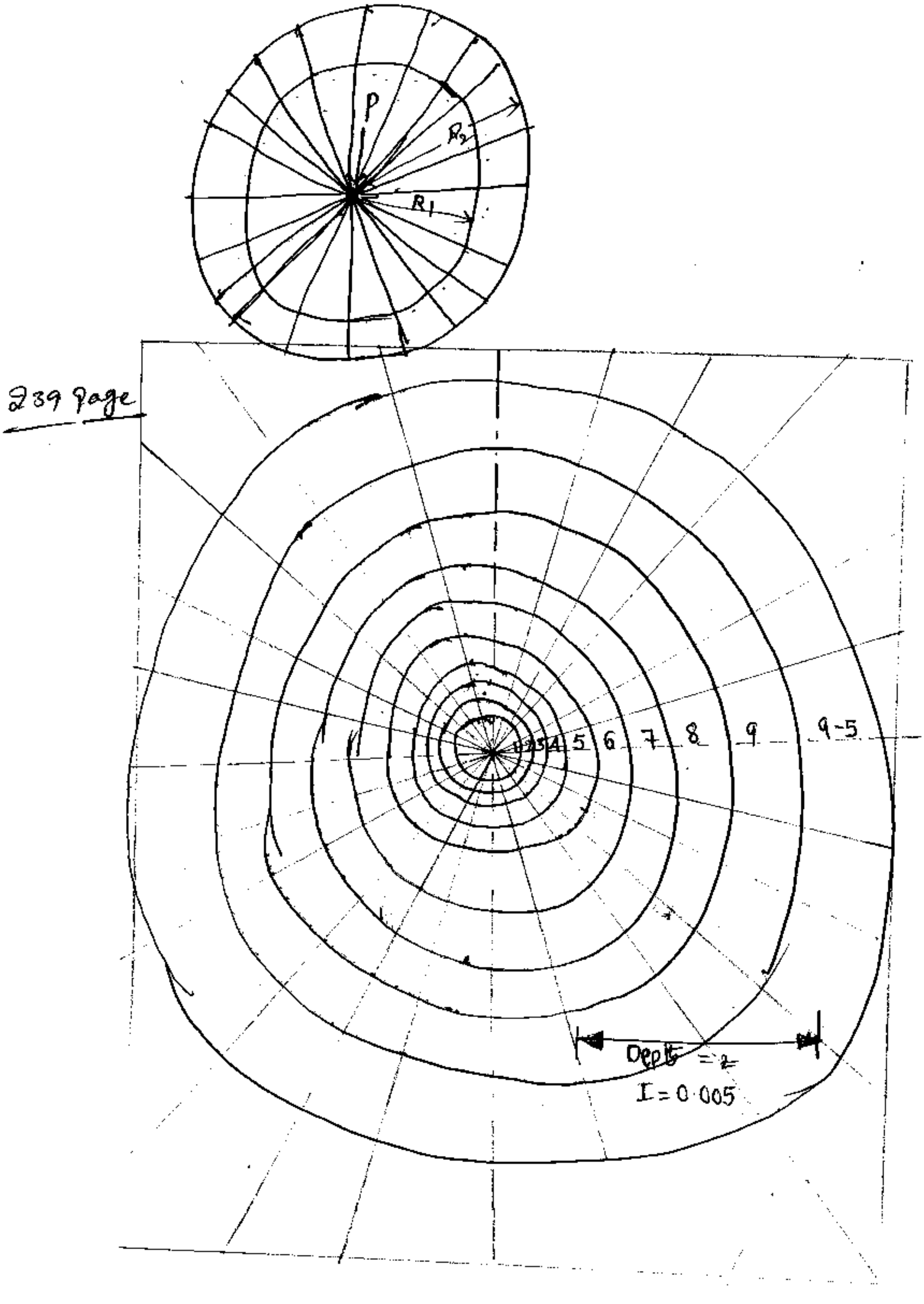
Let us considered another countric circle (11)
of radius R_2 and divide it into 20 equal sectors. Each larger sections is divided into two sub area. If the small are a Q exerts a stress of 0.005q at P. The vertical stress due to both area 1 and area 2 would be equal to $2 \times 0.005q$.

$$2 \times 0.005q = \frac{q}{20}\left[1 - \frac{1}{\left(1+\left(\frac{R}{z}\right)^2\right)^{3/2}}\right]$$

By solving the above expression $\frac{R_2}{z}$ become 0.4

In other words the radius of second circle would be equal to 0.4z. Likewise the radii of the third to the 9th circle can be determined. The values obtained are 0.52z, 0.64z, 0.77z, 0.92z, 1.11z, 1.39z and 1.91z the radius of 9½ circle is 2.54z likewise the radius of 10th circular will become Infinity. There for 10th circle cannot be drawn. Like these newmark's charts are prepared for different Possition ~~differently~~ differently.

while preparing newark charts the co-efficient will multiply with the load is called influence co-efficient In the above case the influence co-efficient is 0.005.



Westergaard's Solution:-

Boussinesq' solution assumes that the soil deposit is isotropic. Actual sedimentary deposits are generally anisotropic. There are generally thin layers of sand embedded in homogeneous clay strata. Westergaard's solution assumes that there are thin sheets of rigid materials sand-wiched in a homogeneous soil mass. These thin sheets are closely spaced and are of infinite rigidity and are, therfore, incompressible. These permit only downward displacement of the soil mass as a whole without any lateral displacement. Therefore, westergaard's solution represents more closely the actual sedimentary deposit.

According to westergaard, the vertical stress at a point p at a depth Z below the concentrated load 'Q' is given by

$$\sigma_z = \frac{C/2\pi}{\left[C^2 + \left(\frac{r}{z}\right)^2\right]^{3/2}} \cdot \frac{Q}{z^2}$$

where c depends upon the poisson ratio(ν) and is given by

$$C = \sqrt{(1-2\nu)/(2-2\nu)}$$

For elastic materials ν ~~ratio~~ varies 0 to 0.5

when ν is zero c will become $\frac{1}{\sqrt{2}}$. then

$$\sigma_z = \frac{\frac{1}{8\sqrt{2}\pi}}{\left[\frac{1}{2}+\left(\frac{\sqrt{2}r}{cz}\right)^2\right]^{3/2}} \times \frac{Q}{z^2}$$

$$\sigma_z = \frac{1}{\pi\left[1+2\left(\frac{r}{z}\right)^2\right]^{3/2}} \cdot \frac{Q}{z^2}$$

$$\boxed{\sigma_z = I_W \frac{Q}{z^2}}$$

where I_W is known as Westergaard influence coefficient

$$\boxed{I_W = \frac{1}{\pi\left[1+\left(\frac{2r}{z}\right)^2\right]^{3/2}}}$$

The values of I_W are considerably smaller than the Boussinesq influence factor (I_B).

Problems

1) A countrated load of 40KN is applied vertically on a horizontal ground surface. Determine the vertical stress intensities at the following points.

(i) At a depth of 2m below the point of application of the load

(ii) At a depth of 1m and at a radial distance of 3m from the line of action of the load

(iii) At a depth of 3m and at a radial distance of 1m from the line of action of the load.

Sq) The Boussinesq's solution $\sigma_z = \frac{3}{2\pi}\frac{Q}{z^2}\left(\frac{1}{(1+(r/z)^2)^{5/2}}\right)$

$Q = 40\,kN = 4\ tones$

(i) $r=0 \quad z = 2m$

$\frac{r}{z} = 0$

$\sigma_z = \frac{3}{2\pi} \times \frac{40}{2^2} \cdot \frac{1}{1^{5/2}} = 4.77\ kN/m^2$

(ii) $z = 1m,\ r = 3m$

$\frac{r}{z} = 3$

$\therefore \sigma_z = \frac{3}{2\pi} * \frac{40}{1^2} \cdot \frac{1}{(1+9)^{5/2}} = 0.06\ kN/m^2$

(iii) $z = 3m,\ r = 1m$

$\frac{r}{z} = \frac{1}{3} = 0.333$

$\therefore \sigma_z = \frac{3}{2\pi} \times \frac{40}{3^2} \times \frac{1}{(1+0.33^2)^{5/2}} = 1.63\ kN/m^2$

2) A rectangular footing 2m×3m in size has to carry a uniformly distributed load of 100 kN/m². plot the distribution of vertical stress intensity on a horizontal plane at a depth of 2m below the base of the footing by the following methods. and compare those two distribution

(i) Boussinesq's Solution

(ii) Two to one dispersion method.

Sq) (a) load = 100 kN/m²

Point load $Q = 100 \times 2 \times 3$

$= 600$ kN

$$\sigma_z = \frac{3}{2\pi} \cdot \frac{Q}{z^2} \left(\frac{1}{(1+(r/z)^2)^{5/2}} \right)$$

$$= \frac{3}{2\pi} \cdot \frac{600}{2^2} \left(\frac{1}{(1+(r/2)^2)^{5/2}} \right)$$

(i) Boussinesq's $r=0$, $\sigma_z = 71.61$ kN/m²

Put $r=1$ $\sigma_z = 40.99$ kN/m² $r=-1$, $\sigma_z = 40.99$ kN/m²

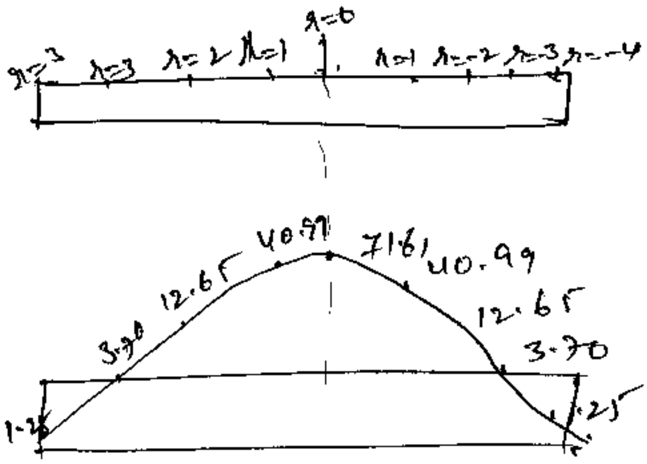
$r=2$ $\sigma_z = 12.65$ kN/m² $r=-2$, $\sigma_z = 12.65$ kN/m²

$r=3$, $\sigma_z = 3.76$ kN/m² $r=-3$, $\sigma_z = 3.76$ kN/m²

$r=4$, $\sigma_z = 1.28$ kN/m² $r=-4$, $\sigma_z = 1.28$ kN/m²

(ii) 2:1 despersion method

$$\sigma_z = \frac{q(B \times L)}{(B+z)(L+z)} = \frac{100 \times 2 \times 3}{(2+2)(2+3)} = 30 \text{ kN/m}^2$$



3) concentra load of 200kN is applied at the ground surface. Determin the vertical stress at a point P which is 6m directly below the load. Also calculate the vertical stress at a point R which is at a depth of 6m but a horizontal distance of 5m from the axis of the load.

So) load intensity $Q = 200kN$

$z = 6m$

$r = 5m$

Vertical stress $\sigma_z = \frac{3}{2\pi}\frac{Q}{z^2} \frac{1}{(1+(r/z)^2)^{5/2}}$

$= \frac{3 \times 2000}{2 \times 3.142 \times 6^2} \frac{1}{(1+(\frac{5}{6})^2)^{5/2}}$

$= 7.096 \ kN/m^2$

P is directly below the load then

$z = 6m, \ r = 0m$

$\sigma_z = \frac{3 \times 2000}{2 \times 3.142 \times 6^2} \times \frac{1}{(1+(\frac{0}{6})^2)^{5/2}}$

$= 26.52 \ kN/m^2$

4) There is a line load of 120 kN/m acting on the ground surface along y-axis, Determine the vertical stress at Points P, Q whose (x, z) ordinates are as follow.

$$P(x, z) = (2, 3.5)$$
$$Q(x, z) = (3, 4.5)$$

So.) For line load $\sigma_z = \dfrac{2q'}{\pi z\left(1+\left(\frac{x}{z}\right)^2\right)^2}$

$P(x, z) = (2, 3.5)$, $q = 120$ kN/m $= q'$

$$\sigma_z = \frac{2 \times 120}{\pi \times 3.5\left(1+\left(\frac{2}{3.5}\right)^2\right)^2}$$

$$= 12.4 \text{ kN/m}^2$$

$$Q(x, z) = (3, 4.5)$$

$$\sigma_z = \frac{2 \times 120}{\pi \times 4.5\left(1+\left(\frac{3}{4.5}\right)^2\right)^2}$$

$$= 8.135 \text{ kN/m}^2$$

5) The unit weight of the soil in a uniform deposite of loose Sand is 16.5 kN/m3. Determine the Geostatic stresses at a depth of 2m. Take the coefficient of Static earth pressure $K_0 = 0.50$

So) $\gamma = 16.5\ kN/m^3$

$z = 2m, \quad K_0 = 0.50$

$\sigma_z = \gamma z$

$= 16.5 \times 2 = 33\ kN/m^2$

$\sigma_h = K_0 \sigma_z = 0.50 \times 33 = 16.5\ kN/m^2$

6) Determine the vertical stress at point P which is 3m below and at a radial distance of 3m from the vertical load of 100kN. used both boussinesq and westergard solution and compare the results and comment.

So.) $z = 3m, \quad r = 3m, \quad Q = 100\ kN$

Bossinesq's solution, vertical stress

$$\sigma_z = \frac{3}{2\pi}\frac{Q}{z^2}\left(\frac{1}{1+(r/z)^2}\right)^{5/2}$$

$$= \frac{3 \times 100}{2\pi \times 3^2}\frac{1}{\left(1+(3/3)^2\right)^{5/2}}$$

$$= 0.937\ kN/m^2$$

Westergard solution

$$\sigma_z = \frac{1}{\pi\left(1+2(r/z)^2\right)^{3/2}} \times \frac{Q}{z^2}$$

$$= \frac{1}{\pi\left(1+2(3/3)^2\right)^{3/2}} \times \frac{100}{3^2} = 0.68\ kN/m^2$$

COMPACTION

compaction is a process of changing the state of the soil by expelling the air in the voids.

compaction process is important in study of earthend dam. construction seepage analysis and in stability of solids.

other definistion of compaction is pressing the soil particles close to eachother by mechanical methods this is part of soil stabilization Air during the compaction is expelled from the void space in the soil. Therefore the massdensity is increased. compaction is the process done for improving engineering properties of soil. The compaction process may be accumplished by rolling, tanping, (or) vibration.

compaction is some what different from consolidation. while consolidation is a gradual process of volume reduction under sustained loading, compaction refers to a more or less rapid reduction mainly in the air void under a loading of short duration.

Difference between compaction and consolidation :-

Consolidation	Compaction
(1) consolidation is the process that changes the state of the soil by expelsion of water.	(1) compaction is the process that changes the state of the soil by expelsion of air.
(2). consolidation is a slow process.	(2) compaction is a rapid process.
(3) settlement are slower	(3) settlements are faster.
(4) consolidation occurs in all three dimensions But we study consolidation in one direction only.	(4) Since the boundaries in other direction this confined compaction occurs in 1-D only.
(5) consolidation occurs in field when saturated soils are subiected to static load.	(5) compaction is artificially done to construct em-backment and earthen dams.

Factors affecting the compaction:-

The dry density of the soil is increased by compaction. The increase in dry density depends upon the following factors.

1. Water content:-

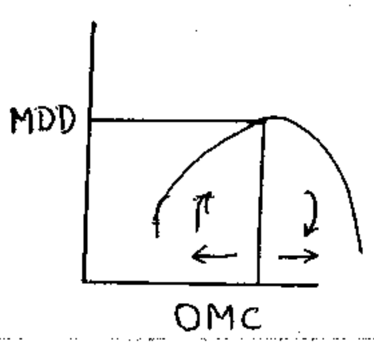
At low water content the soil is stiff and offers more resistence to compaction. As water is increased the soil particles gets lubricated. The soil mass becomes more workable and the particles have closer packing.

The dry density of the soil increases with an increase in water content. Till the optimum water content is reached.

At the stage the air voids attain approximately a constant volume.

Further increase in water content the air voids donot decrease. But the total voids (air+water) increase and the dry density decrease.

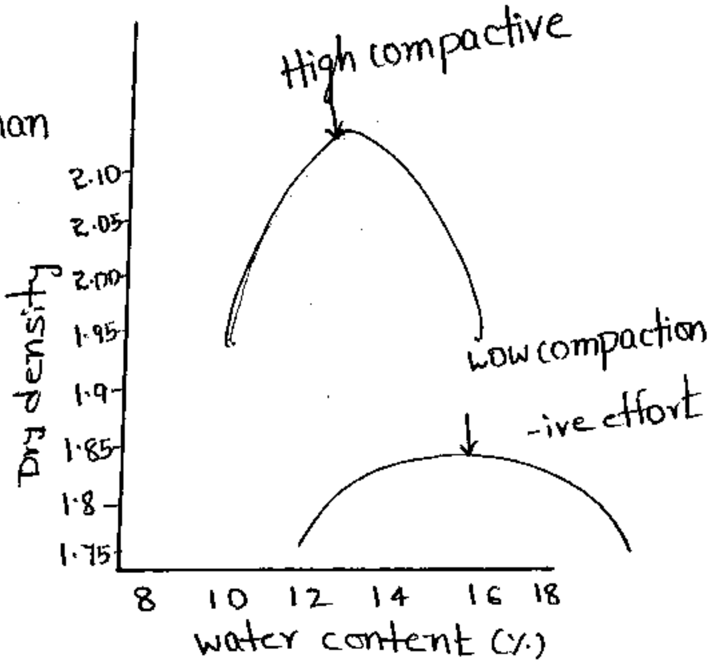
The higher drydensity is achieved upto the optimum water content. Due to forcing in air out of the soil from the soil



After the optimum water content is reached it becomes more difficult to force air out of the soil. Then we have to increase the moisture content to reduce the dry density.

2) Amount of compaction (or) compactive afford :-

The effect of increase in the amount of compactive effort is "to increase the maximum dry density and to reduce the optimum water content. This is shown in figure.



The water content lessthan the optimum the effect of increased compaction is more predominant.

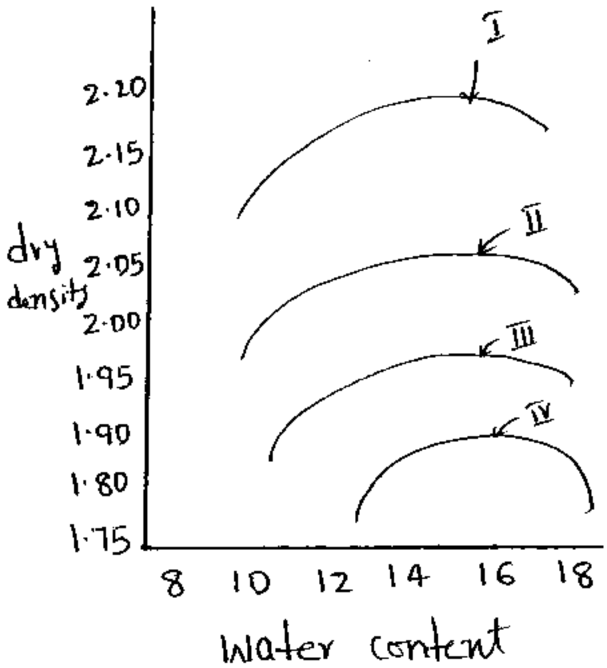
At a water content morethan the optimum the volume of air voids becomes almost constant and the effects of increased compaction is not significant.

From this we can inform that compaction is kevior on dry of optimum and it is lighter on wet of optimum.

3. Type of soil :-

The dry density achived depends on the type of soil. In general coarse grain soil can be compacted to higher drydensity than fine grain soils. with the addition of even a small quantity of fines to a coarse grain soils the soils obtained much higher dry density for the same compactive effort.

I - Well graded sand
II - Low plasticity silt
III - Low plasticity clay
IV - High plasticity clay



4. Method of compaction :-

The dry density achieved depends not only compactive effort but also on the method of compaction.

For the amount of compactive affort the drydensity will depends upon wheather the method of compaction utilised kneading action (or) Dynamic action (or) static action.

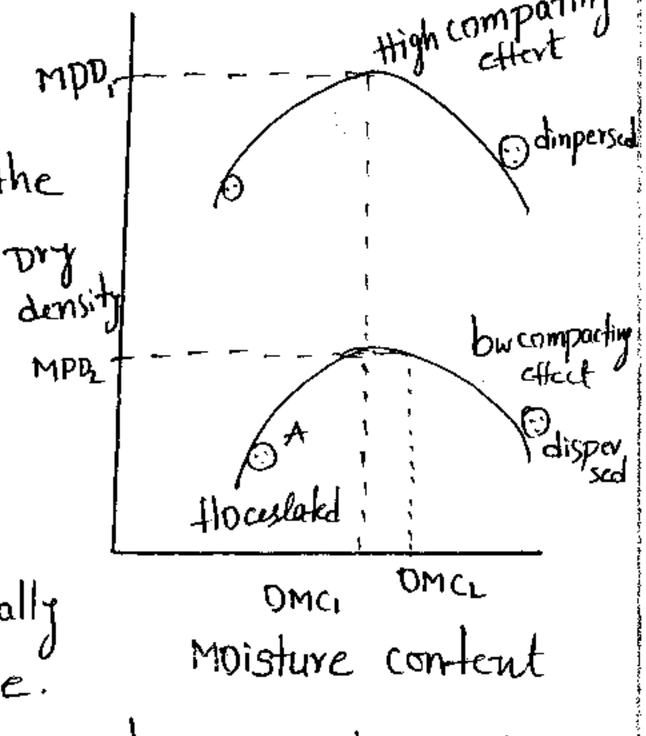
Effect of compaction on properties of soils:-

The engineering property of the soil are improved by compaction. when we are constructing an embankment the desirable properties are achieved by proper selection of type.

The following properties are changed due to compaction of soil. The following discussion the phase "dry of optimum" means the water content is less the optimum moisture content, and the phase 'wet of optimum means the water content is more than O.M.C.

(i) Soil structure:-

The water content in the compacted soils plays an vital role in changing the properties of the soil.



soils compacted at a water content lessthan OMC generally have a floccuated structure.

soils compacted at a water content less than OMC generally have a flocculated structure.

soil compacted at a water content more than the optimum water content usually have a dispersed structure. The structure won't depend on the type of compaction.

If OMC_1 and MDD_1 are the soil parameters at high compacting effort and OMC_2, MDD_2 are the soil parameters at low compacting effort. Then OMC_1 is lesser than OMC_2 and MDD_1 is greater than MDD_2.

$$OMC_1 < OMC_2$$

$$MDD_1 > MDD_2$$

In figure the point A on dry side of the optimum the water content is so low that the attractive force's are more predoments than the repulsive forces. This results in flocculated structure.

2. <u>permeability</u>:- The permeability of a soil depends upon the size of voids. The permeability of soil decreases with an increase in water content on the dry side of the optimum water content. There is an improved orientation of the particles and corresponding reduction in

the size of voids which cause a decrease in permeability. The minimum permeability occurs at or slightly above the optimum water content. After that stage, the permeability, slightly increase, but it always remains much less than that on the dry side of the optimum. The slight increase in the drydensity is more pronounced than the effect of improved orientation.

3) Swelling:- A soil compacted dry of the optimum water content has high water deficiency and more random orientation of particles. Consequently, it imbibes more water than the sample compacted wet of the optimum, and has, therefore more swelling.

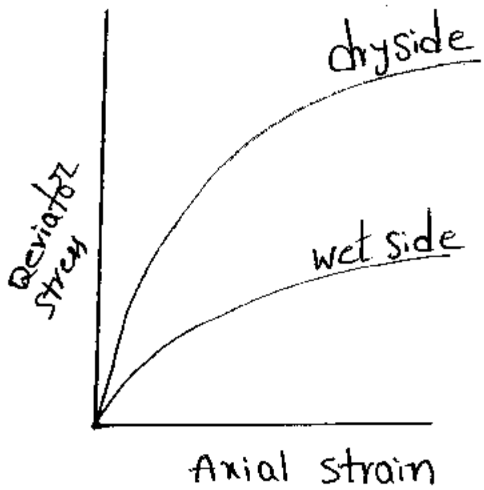
4. pore water pressure:- A sample compacted dry of the optimum has low water content. The pore water pressure developed for the soil compacted dry of the optimum is therefore less than that for the same soil compacted wet of the optimum.

(5) compressibility:- The flocculated structure developes on the dry side of the optimum offers greater resistance to compression than the dispersed structure on the wet side consequently, the soils on the dry side of the optimum are less compressible.

(6) stress-strain Relationship:-

The soils compacted dry of optimum has steeper stress-strain curve.

The modulus of elasticity for the soils compacted dry of optimum is high.



Such soils have brittle failure. Like dense sands and over consolidated soils.

The soils compacted on wet of optimum will having relatively flater stress-strain curve, and it will have lower modulus of elasticity.

The failure in this case occurs at a large strain and it is of plastic in nature.

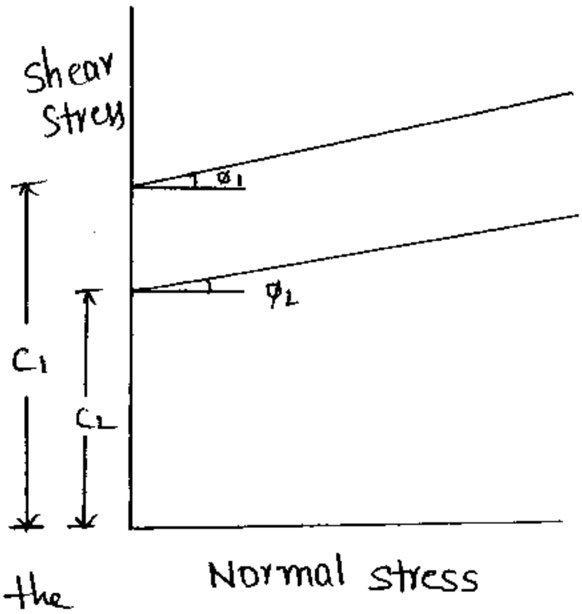
7) Shear strength :-

Since shear strength is a function of wherion and bulk density. The change in water contant changes the bulkdensity so as shear strength.

In general at a given water content the shear strength of the soil increases with an increase in the compactive effort till a critical degree of saturation is reached.

The shear strength of compacted soils depends upon the soil type, the moduler water content, drainage conditions and method of compaction.

The soil compacted dry of optimum have a higher shear strength at low strains. However, at large strain the flocculated structure for the on the dry side in broken and the ultimate strength is approximately equal for both samples.



On wet side the shear strength is further reduced. If compaction is reduced.

Problems

(1) The OMC of a soil is 16.5% and its maximum dry density is 1.57 g/cc. The specific gravity of solids is 2.65. Determine (i) The degree of saturation and percentage of air voids of the soil at OMC. (ii) Theoritical dry density at O.M.C corresponding to zero air voids.

Sol:- (i) $\gamma_d = 1.57 \, g/cc$

$\omega = 16.5\%$, $G = 2.65$

$$\gamma_d = \frac{G\gamma_w}{1 + \frac{WG}{S}}$$

$$eS = WG$$

$$e = \frac{WG}{S}$$

$$1.57 = \frac{2.65 \times 1}{1 + \frac{165 \times 2.65}{S}}$$

$$1 + \frac{0.165 \times 2.65}{S} = \frac{2.65}{1.57}$$

$$\frac{0.437}{S} = 0.687$$

$$S = 0.636$$

$$= 63.6\%$$

The soil at OMC has a moisture of 16.5% and dry density of 1.57 g/cc.

percentage of air voids $= 100 - S$

$= 100 - 63.6$

$= 36.4\%$

(ii) At zero air voids the soil is fully saturated

i.e $S=1$

$$\gamma_d = \frac{G\gamma_w}{1+e}$$

$$Se = WG$$

$$e = WG \quad (\because S=1)$$

$$\gamma_d = \frac{G\gamma_w}{1+WG} = \frac{2.65\times 1}{1+0.165\times 2.65}$$

$$\gamma_d = 1.843 \text{ g/cc.}$$

(2) During the construction of an embankment the density attained by field compaction was investigated by sand jar method. A test pit was excavated in the newly compacted soil and was filled up by pouring sand. The following observations were made. weight of soil excavated from pit equal to 2883g. Bulk density of sand equal to 1.52g/cc. Moisture content of embankment soil = 16%. Determine the dry density of the compacted soil.

Sol: The volume of sand required to fill up the pit

$$= \text{volume of pit} = \frac{\text{wt. of sand}}{\text{bulk density of sand}}$$

$$V = \frac{2883}{1.52}$$

$V = 1897 \, cm^3$

Wt of the soil excavated from the pit = 2883g

$$\gamma = \frac{\text{wt. of soil}}{\text{volume}} = \frac{2883}{1897} = 1.51 g/c.c \qquad W = 16\% = 0.16$$

$$\therefore \text{Dry density} = \gamma_d = \frac{\gamma}{1+w} = \frac{1.51}{1+0.16} = 1.30 g/cc.$$

3. It is reauired to construct an embackment by compacting a soil excavated from near by barrow area. The optimum moisture content and maximum drydensity of the soil for determine in the laboratory and were found to be 22.5% and 1.66g/cc respectively. However, the natural moisture content and bulkdensity of soil were 9% and 1.75g/cc respectively. Final out the quantity of the soil to be excavated and the quantity of water to be added to it for every $100m^3$ of finished embankment.

Sol: The embankment should be constructed by compacting the soil obtained from barrow area which is at the OMC and corresponding dry density. But the natural moisture content of the existing soil is lessthan its OMC. Hence a certain amount of water is to be added to the soil prior to the compaction.

Maximum dry density is given as 1.66 g/cc

$$= \frac{1.66 \times 10^{-3} \times 10^{-3} t}{10^{-6}}$$

$= 1.66$ tone/m^3

$[1 g/cc = 1 t/m^3]$

$\left[1 g/cc = \frac{10^{-3} g}{10^{-6} m^3}\right]$

$$= \frac{10^{-3} \times 10^{-3}}{10^{-6} m^3}.$$

But $\gamma_d = \frac{W_d}{V} \Rightarrow W_d = \gamma_d V.$

In the problem 1 unit of embakment is given as $100 m^3$ of finished embankment.

From this wt. of dry soil for $100 m^3$ embankment. $W_d = 1.66 \times 100 = 166$ tones.

The weight of water reauired to get the optimum moisture content.

$$W = \frac{W_w}{W_d}$$

$$W_w = W \times W_d$$

$$= 0.225 \times 166$$

$$= 37.35 \text{ tones.}$$

But the Bulk of the

But the bulk density of the existing soil (or) transported soil = 1.78 g/cc = 1.78 t/m³. and the moisture content is 9%.

$$\gamma_d = \frac{\gamma}{1+w}$$

$$= \frac{1.78}{1+0.09} = 1.633\ t/m^3$$

The volume of soil V_b to be obtained from barrow area in order to obtained 166 tones of dry soil.

$$V_b = \frac{\text{wt. of dry soil excavated}}{\gamma_d \text{ in natural in condition.}}$$

$$V_b = \frac{166}{1.633} = 101.65\ m^3$$

volume of extra soil to be added = $1.65\ m^3$

weight of the water avilable from the soil in natural condition = wt. of dry soil × Natural water content

$= 166 \times 0.09 = 14.94$ tones

weight of water avilable per unit embankment = 37.35 tones.

quantity of water to be added = 37.35 − 14.94

= 22.41 tones.

volume of water to be added = $\frac{\text{wt. of water}}{\text{density of water}}$

$= \frac{22.41}{1\ t/m^3} = 22.41\ m^3$

we know that 1 litre $= 10^{-3}\ m^3$

$1\ m^3 = 1000$ lts.

$22.41\ m^3 = 22.41 \times 1000$ lts

$= 22410$ lts.

To construct $100\ m^3$ of embankment $101.65\ m^3$ of soil is to be excavated from the barrow pit and 22410 litres of water is added.

7. An embankment was constructed by compacting at a moisture content of 15.5% and dry density of 1.72 g/cc. If the specific gravity of solid soils be 2.68 Determine the void ratio and degree of saturation of embankment of soils.

Sol:- $W = 15.5\% = 0.155$

$G = 2.68$

We have $\gamma_d = 1.72$

$$\frac{G\gamma_w}{1+e} = 1.72$$

$$1+e = \frac{2.68 \times 1}{1.72} = 1.558$$

$$e = 0.558$$

$$Se = WG$$

$$S = \frac{WG}{e} = \frac{0.155 \times 2.68}{0.558} = 0.744 = 74.4\%.$$

The required degree of saturation $= 74.4\%$.

5. The rock contant in a filled is 80% dry weight. The rock can be compacted to a minimum void ratio of 0.73. The man dry unit weight to which the soil fraction can be compacted is 1.63g/cc What is the maximum dry density to which the fill can be compacted Given specific gravity of rock is 2.56.

Sol:- When the rock present in the fill is compacted to the densest state its dry unit weight is given by

$$\gamma_{dmax} = \frac{G\gamma_w}{1+e_{dense}}$$

$$\gamma_{dmax} = \frac{2.56\times 1}{1+0.73} = 1.479\,g/cc$$

For the soil $\gamma_{dmax} = 1.63$ (Given in problem)

Let us now consider 1unit soil of the given fill (1g of soil is taken as unit in fill)

According to the question the weight of soil and rock present in the given sample are 0.2g and 0.8g respectively.

Now volume of 0.8g of rock $= \frac{\text{weight}}{\text{density}}$

$= \frac{0.8}{1.48} = 0.54\,cc$

volume of 0.2g of dry soil $= \frac{0.2}{1.63} = 0.122\,cc.$

Total volume of 1g of fill $= 0.54 + 0.112$

$= 0.662\,c.c$

Maximum dry density of composite fill

$= \frac{\text{Dry weight}}{\text{volume}}$

$= \frac{1\,gm}{0.662} = 1.508\,g/cc.$

6) In order to determine the relative density of sand sample, its natural moisture content and bulkdensity were determined in the field, and were found to be 7% and 1.61g/cc respectively. Samples of this soil were then compacted in proctor's mould of 1/30 cubic feet capacity at loosest and densest state the following data were obtained.

weight of the empty mould = 2100g

weight of the mould + soil in the loosest state: 3363.6gm

weight of mould + soil in the densest state:

= 3857.4g

moisture content of the sample used in test = 11%.

Determine the relative density of the sand and coment on its type.

Sol: volume of the mould $= \frac{1}{30}$ cu.ft

$$= \frac{1 ft^3}{30} = \frac{(12)^3 \times 2.54^3}{30}$$

$$= 943.89 \text{ c.c}$$

The bulk density in the loosest state

$$\gamma_1 = \frac{3363.6 - 2100}{943.89}$$

$$= 1.339 \text{ g/cc}$$

Dry density in the loosest state $= \gamma_{d\,min}$

$$\gamma_{d\,min} = \frac{\gamma}{1+w} = \frac{1.339}{1+0.11} = 1.206 \text{ g/cc}$$

Bulk density in the densest state $= \frac{3857.4 - 2100}{943.89}$

$$\gamma_2 = 1.861 \text{ g/cc}$$

Dry density in the denses state $= \gamma_{d\,max}$

$$\gamma_{d\,max} = \frac{1.861}{1+0.11} = 1.67 \text{ g/cc}$$

Natural bulk density is given as

$$\gamma = 1.61 \text{ g/cc}$$

and natural moisture content is given as 7%.

∴ Insitu dry density (or) Natural dry density

$$\gamma_d = \frac{1.67}{1+0.07} = 1.504 \text{ g/cc}$$

$$\text{Relative density} = \frac{\gamma_{d\,max}}{\gamma_d} \times \frac{\gamma_d - \gamma_{d\,min}}{\gamma_{d\,max} - \gamma_{d\,min}}$$

$$= \frac{1.67}{1.504} \times \frac{1.504 - 1.206}{1.67 - 1.206}$$

$$= 0.713 = 71.3\%$$

Field compaction Methods/Equipment:-

Several methods are used for compaction of soil in field. The choice of the method will depend upon the soil type, the maximum dry density required, and economic consideration. Some of the more commonly used conventional methods are disscussal below.

1) Tampers:- A hand-operated tamper (or rammer) consists of a block of iron (or stone), about 3 to 5 kg in mass, attached to a wooden rod. The rammer is lifted for about 0.30m and dropped on the soil to be compacted. A mechanical rammer is operated by compressed air gasoline

power. It is much heavier, about 30 to 150kg. Mechanical rammers have been used upto a mass of 1000kg in some special cases.

Tampers are used to compact soil adjacent to existing structure or confined areas, such as trenches and behind the bridge abutments where other methods of compaction cannot be used. Owing to very low output, tampers are not economical where large quantities of soils are involved. Tampers can be used for all types of soils.

(2) Rollers:- Rollers of different types are used for compaction of soils. The compaction depends upon the following factors.

(i) contact pressure:- In general, the compaction increases with an increase in the contact pressure. For a smooth-wheel roller, the contact pressure, pressure depends upon the load per unit width and the diameter of the roller.

(ii) Number of passes:- The compaction of a soil increases with an increases in the number of passes made. However, beyond a certain limit, the increases in the density with an increase in the number of passes in not applicable. From economy consideration, the number of passes generally restricted to a reasonable limit between 5 to 15.

(iii) Layer thickness:- The compaction of a soil increases with a decrease in the thickness of the layer. However, for economy considaration, the thickness is rarely kept lessthan 15cm.

(iv) Speed of roller:- The compaction depends upon the speed of the roller. The speed should be so adjusted that the maxi-mum effect is achieved.

Types of Rollers:-

(a) Smooth-wheel rollers:-

A smooth wheel rollers generally consists of three wheels, two large wheels in the rear and one small wheel in the front. A tandem type smooth wheel roller consists of only two drums; one in the rear and one in the front. The mass of a smooth wheel roller generally varies between 2 to 15 Mg. These roller are operated by internal combustion engines.

(b) Rubber tyred roller:- (or) (pneumatic-tyred roller)

The maximum weight of this roller may reach 2000 kN. The smaller rollers usually have 9 to 11 tyres on two axles with the tyres spaced so that a complete coverage is obtained with each pass. The tyre loads of the smaller roller are in the 7.5 kN and the tyre pressure in the oder of 200 kN/m^2. The large rollers have tyre loads ranging from 100 to 500 kN per tyre, and tyre pressure range from 400 to 1000 kN/m^2.

(c) Sheep foot roller:-

Sheep foot rollers are avilable in drum widths ranging from 120 to 180cm and in drum diameters ranging from 90-180cm. projections like a sheeps foot are fixed on the drums. The lengths of these projection range from 17.5cm - 23cm. The contact area of the tamping foot ranges from 35 to 56cm². The loaded weight per drum ranges from about 30KN for the smaller sizes to 130kg| 130KN for the layer sizes.

(d) Vibratory roller:-

The weight of vibratory rollers range from 120 to 300kN. In some units vibration is produced by weights placed eccentrically on a rotating shaft in such a manner that the forces produced by the rotating weights are essentially in a vertical direction. vibratory rollers are effective for compacting granular soils.

UNIT-VII
CONSOLIDATION

Introduction:-

When a soil mass is subjected to a compressive force, like all other materials, its volume decrease. The property of the soil due to which a decrease in volume occurs under compressive forces is known as the compressibility of soil.

The compression of soil can occur due to one or more of the following causes.

1. compression of solid particles and water in the voids.
2. compression and expulsion of air in the voids.
3. Expulsion of water in the voids.

The compression of a saturated soil under a steady static pressure is known as consolidation. It is entairly due to expulsion of water from the voids.

The consolidation of a soil deposit can be divided into 3 stages.

1. Initial consolidation:- when a load applied to a partially saturated soil, a decrease in volume occurs due to expulsion and compression of air

in the voids. A small decrease in volume occurs due to compression of solid particles. The reduction in volume of the soil just after application of load is known as initial consolidation (or) initial compression.

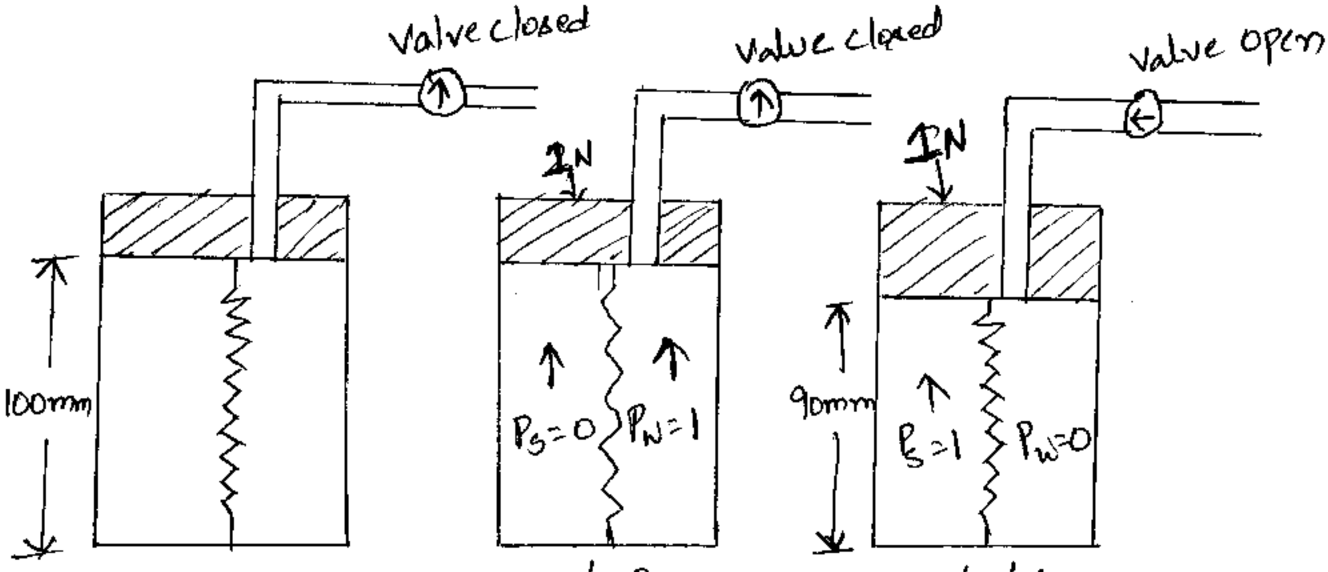
2. Primary consolidation:- After initial consolidation, further reduction in volume occurs due to expulssion of water from voids. When a saturated soil is subjected to a pressure, initially all the applied pressure is taken by water as an excess pore water pressure, as water is almost incompressible as compared with solid particles. A hydraulic gradient develops and the water starts flowing out and a decrease in volume occurs. This redution in volume is called primary consolidation.

3. Secondary consolidation:- The reduction in volume continuous at a very slow rate even after the excess hydrostatic pressure developed by the applied pressure is fully dissipated and the primary consolidation is complete.

This additional reduction in the volume is called Secondary consolidation.

SPRING ANALOGY FOR PRIMARY CONSOLIDATION :-

The process of primary consolidation can be explained with the help of the spring analogy given by Terzaghi.



(a) $t=0$ (b) $t=t_f$ (c)

The above fig shows a cylinder fitted with a tight-fitting piston having a valve. The cylinder is filled with water and contains a spring of specified stiffness. Let the initial length of the spring is weightless and and the spring and water are initialy free of stress.

When a load P (say, 1N) is applied to the piston, with its valve closed, the entire load taken by water (fig(b)). The stiffness of spring be 10 mm/N. Let us assume that the piston is weightless and the spring and water are initially free of stress.

~~When a load P (sa~~

the spring is negligible compared with that of the spring is taken by spring. water, and consequently, no load is taken by spring.

From equilibrium,

$$P_W + P_S = P \longrightarrow (1)$$

where P_W = load taken by water,

P_S = load taken by spring

P = total load.

for $P = 1\,N$, The above equation becomes

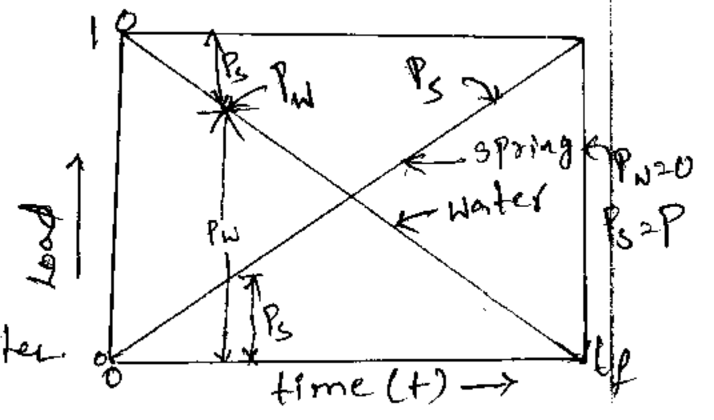
$$P_W + P_S = 1 \longrightarrow (2)$$

Initially ($t=0$) when valve is closed $P_S = 0$, Therefore $P_W = 1$

If the valve is now gradually opened, water starts escaping from the cylinder. The spring starts sharing some load and a decrease in its length occurs when a portion (ΔP) of the load is transferred from the water to the spring, The the equation (2) becomes

$$\Delta P + (1.0 - \Delta P) = 1.0$$

As more and more water escapes, the load carried by the spring increases. The figure shows the transfer of the load from spring to water.



Pressure - void ratio curves (e-p curves): for clays :-

The compressibility characteristics of clays depen on many factors. The most important factors are:

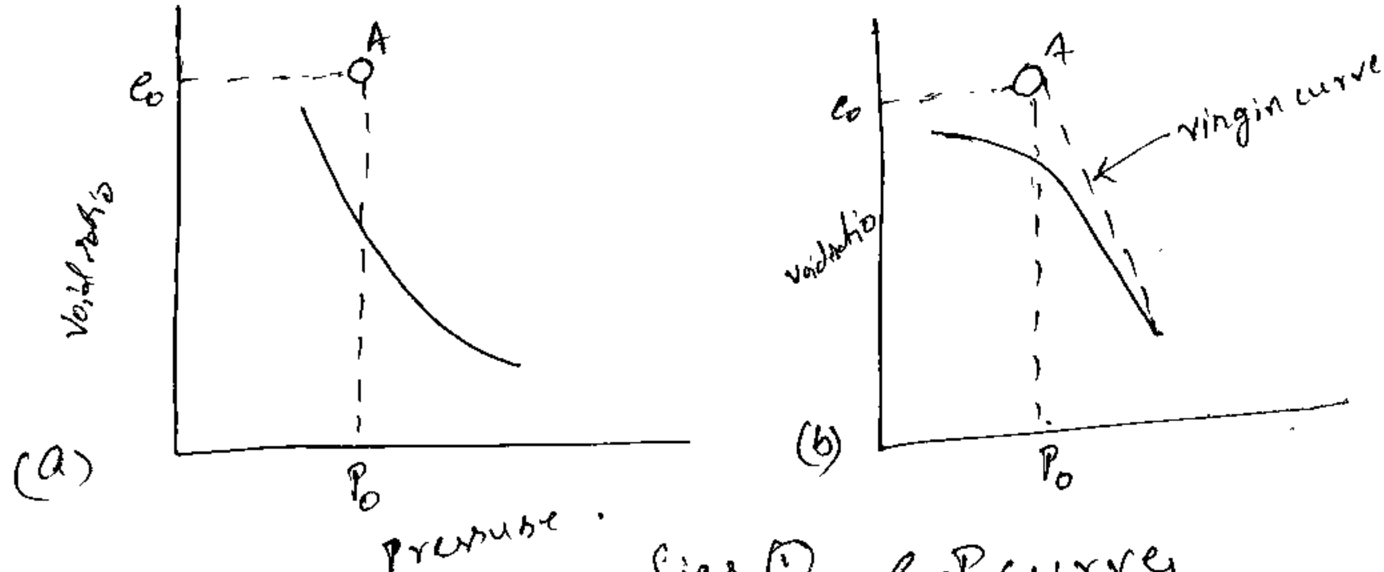
1. whether the clay is normally consolidated or over consolidat
2. whether the clay is sensitive or insensitive.

A clay is said to be normally consolidated if th present effective overburden pressure P_0 is the maxim pressure to which the layer has ever been subjected at any time in its history, whereas a clay layer is sai to be over consolidated if the layer was subjected at one time in its history to a greater effective over - burden pressure, P_c, than the present pressure, P_0. The ratio $\frac{P_c}{P_0}$ is called the over consolidation ratio (OCR).

The over consolidation of a clay stratum may have been caused due to some of the following factors.

1. Due to weight of an over burden of soil which has eroded.
2. Due to weight of continental ice sheet that melted.
3. Due to desiccation of layer close to the surface.

Experience indicates that the natural moisture content W_n, is commonly close to the liquid limit, W_L, for normally consolidated clay soil whereas for the over-consolidated clay, W_n is close to plastic limit W_p.



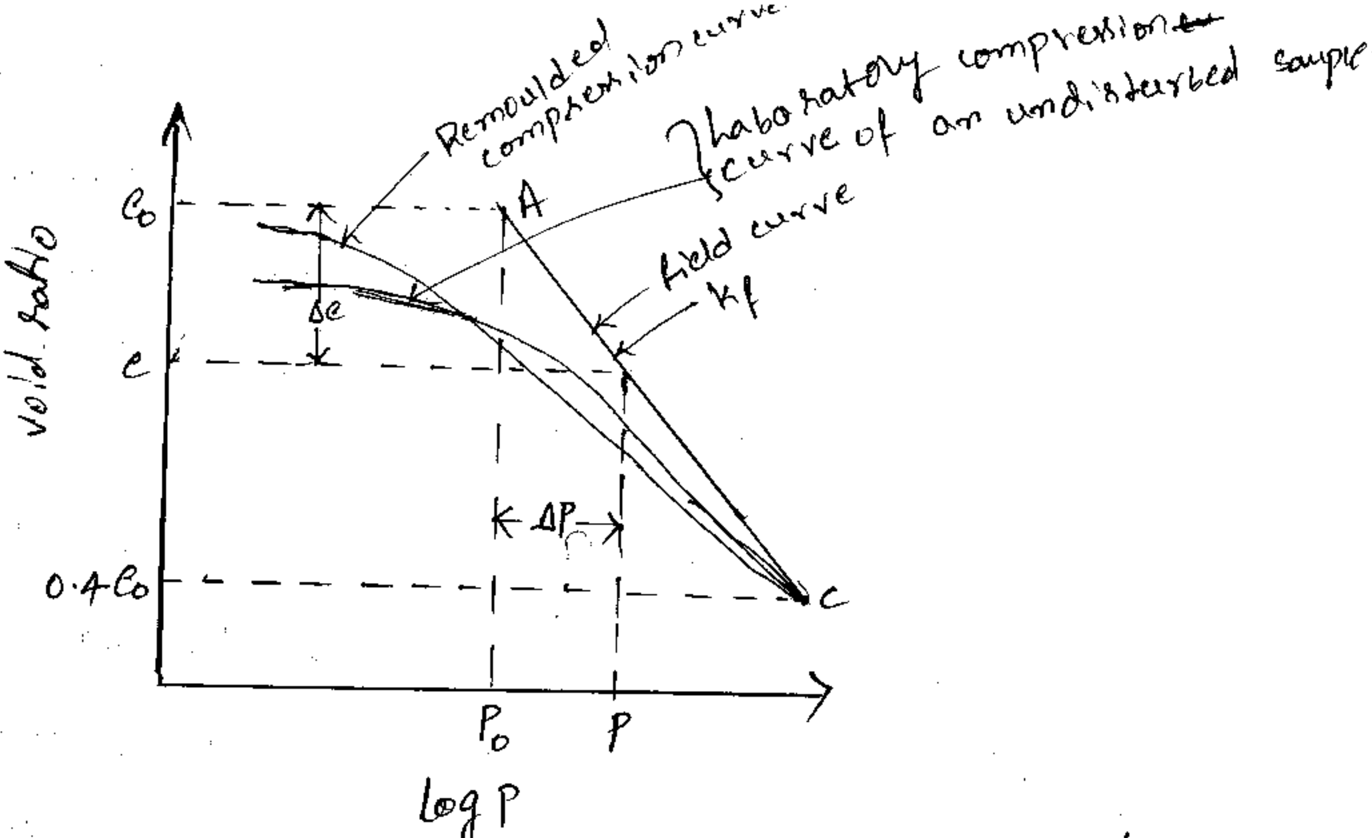
figs (1) e-P curve

e log - P curve

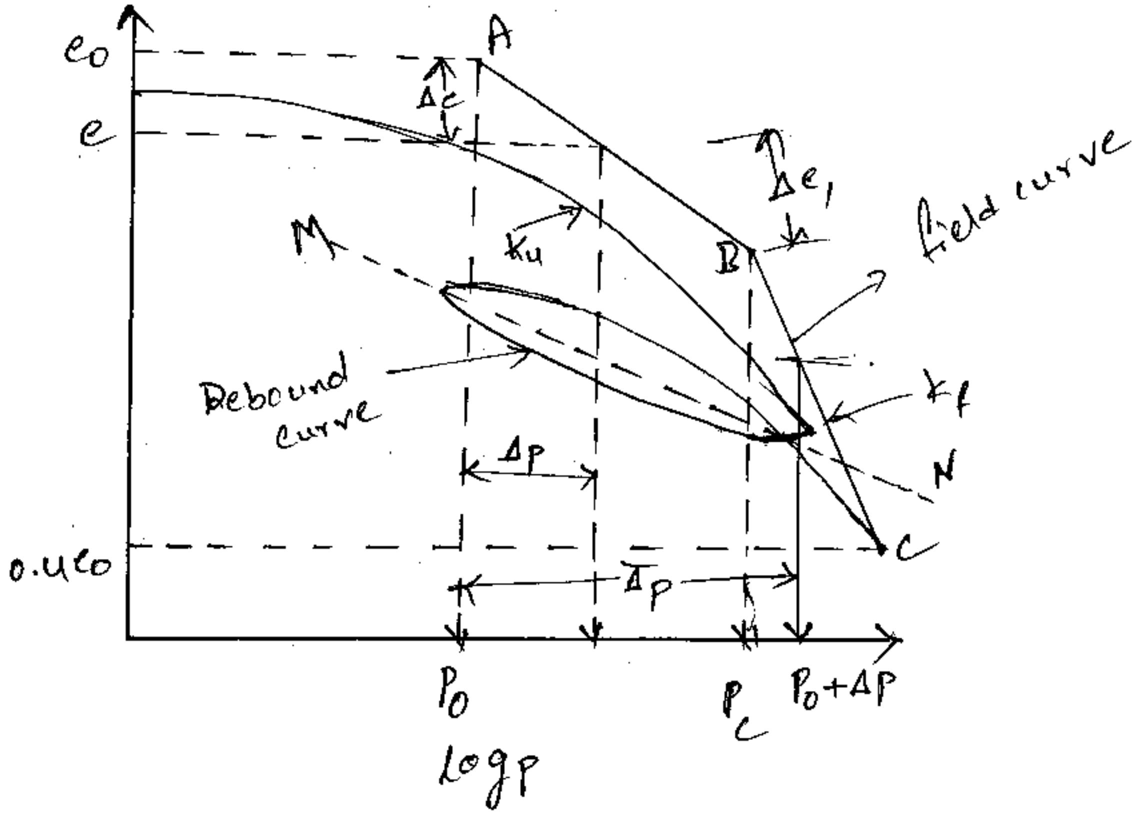
It has been explained earlier with reference to the above figures (1) that the laboratory e-log P curves of an undisturbed sample does not pass through Point A and always passes below the point. It has been found from investigation that the inclined straight portion of e-logP curves of undisturbed or remoulded samples of clay soil intersect at one point at a low void ratio and corresponds to $0.4e_0$ shown as Point C in the fig (2). It is therefore, logical to assume the field curve labelled as K_f should also pass through this point

The field curve can be drawn from point A, having co-ordinates (e_0, P_0) which corresponds to the in-situ condition of the soil. The straight line AC in fig (2a) gives the field curves K_f for normally consolidated clay soil of low sensitivity.

The field curve for over consolidated clay soil cons of two straight lines, represented by AB and BC in fig (2b). Schmertmann (1955) has shown that the initial section AB of the field curve is parallel to the mean slope slope MN of the rebound labora curve. Point B is the intersection point of the vertical line passing through the per consolidation pressure Pc on the abscissa and the sloping line AB. Since point C is the intersection of the laborat compression curve and the horizontal line at void ratio $0.4e_0$, line BC can be drawn. The slope of line MN which is the slope of the rebound curve is called the swell index C_s.



(a) Normally consolidated clay soil



(b) Preconsolidated clay soil.

Fig - 2 :- e-log P curve

Magnitude and rate of consolidation:-

It has been explain that the ultimate settlement S_t of a clay layer due to consolidation may be computed by using the following two equations

$$S_t = \frac{C_c}{1+e_0} H \log_{10} \frac{P_0 + \Delta P}{P_0} \longrightarrow (1)$$

where S_t is ultimate settlement

C_c is compression index.

$$S_t = \Sigma H_i \frac{C_c}{1+e_0} \log_{10} \frac{P_0 + \Delta P}{P_0}$$

If S is the settlement at any time t after the imposition of load on the clay layer, the degree of consolidation of the layer in time t may be expressed as

$$U\% = \frac{S}{S_t} \times 100 \%.$$

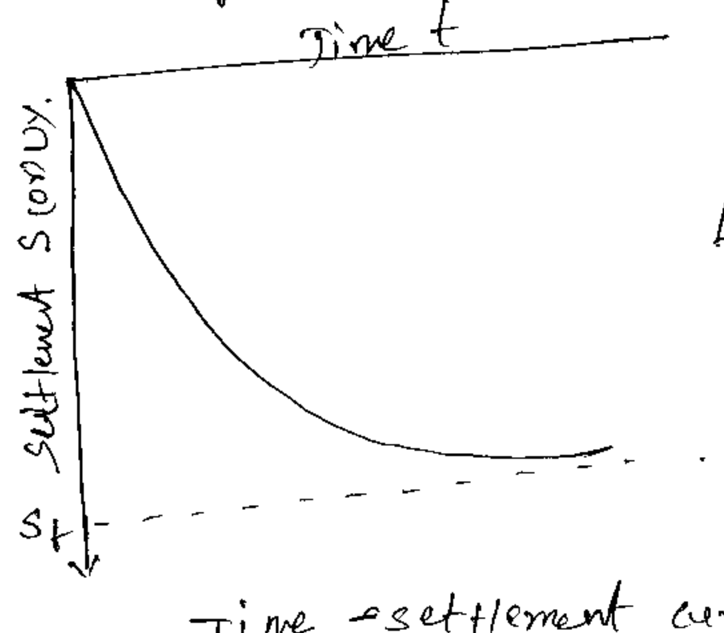
Since U is a function of the time factor T, we may write

$$U\% = 100 f(T) = \frac{S}{S_t} \times 100$$

The rate of settlement curve of a structure built on a clay layer may be obtained by the following procedure.

1. From consolidation test data, compute m_v and c_v
2. compute the total settlement S_f that the clay stratum would experience with the increment of load ΔP.

   { c_v = co-eff of volume compressibility }
   { $m_v = \frac{a_v}{1+e_0}$ " " " " }
3. From the theoretical curve giving the relation between U and T, find T for different degrees of consolidation, say 5, 10, 20, 30 percent etc.
4. compute from equation $t = \frac{TH^2}{c_v}$ the values of t for different values of T. It may be noted here that for drainage on both sides H is equal to half the thickness of the clay layer
5. Now a curve can be plotted giving the relation between t and U% or t and s as shown in the below fig.



Time-settlement curve.

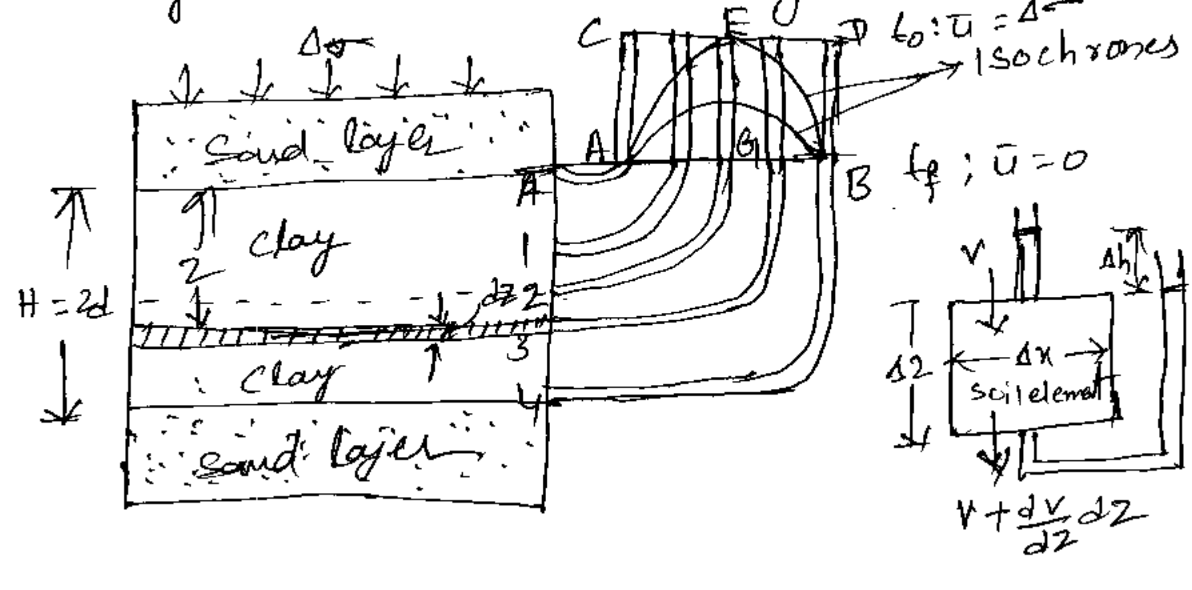
a_v = coeff of compressibility

Δp = net change in pressure

TERZAGHI'S THEORY OF CONSOLIDATION:-

Assumptions:- Terzaghi (1925) gave the theory for the determination of the rate of consolidation of a saturated soil mass subjected to a static, steady load. The theory is based on the following assumption.

1. The soil is homogeneous and isotropic
2. The soil is fully saturated.
3. Soil particles and water are incompressible.
4. Darcy's law is valid throughout the consolidation process
5. coefficient of permeability is constant during consolidation.
6. Excess pore water drains out only in the vertical direction.
7. The time lag in consolidation is due to entirely to the low permeability of the soil
8. The change in thickness of the layer during consolidation is insignificant



The above figure shows a clay layer of thickness H, sandwhich between two layers of sand which serves as drainage faces. when the layer is subjected to a pressure increment $\Delta\sigma$, excess hydrostatic pressure is set up in the clay layer. At the time t_0, the instant of pressure application, whole of the consolidati Pressure $\Delta\sigma$ is carried by the Pore water so that the Initial excess hydrostatic Pressure $\bar{U}_0$ is equal to $\Delta\sigma$, and is represented by a straight line $\bar{u}=\Delta\sigma$ on the pressure distribution diagram. As water starts escaping into the sand, the excess hydrostatic Pressure at the pervious boundaries drops to zero and remains so at all times. After a very great time t_f, the whole of the excess hydrostatic Pressure is dissipated so that $\bar{u}=0$, represented by line AGB. At an intermediate time t, the consolidating pressure $\Delta\sigma$ is partly carried by water and partly by soil, and the following relationship is obtained : $\Delta\sigma = \Delta\sigma' + \bar{u}$. The distribution of excess hydrostatic pressure $\bar{u}$ at any time 't' is Indicated by the curve AFB, Joining water levels, in the piezometric tubes ; ~~this curve~~ is this curve is known as isochrone, and number of such isochrones can be drawn at various time intervals t_1, t_2, t_3 etc. The slope of isochrones at any point at a given time indicates the rate of change of $\bar{u}$ with depth.

At any time t, the hydraulic head h corresponding to the excess hydrostatic pressure is given by

$$h = \frac{\bar{u}}{\gamma_w} \longrightarrow ① \quad \left\{ \because \frac{\partial h}{\partial z} = \frac{1}{\gamma_w} \frac{\partial \bar{u}}{\partial z} \right\}$$

Hence the hydraulic gradient i is given by

$$i = \frac{\partial h}{\partial z} = \frac{1}{\gamma_w} \frac{\partial u}{\partial z} \longrightarrow ②$$

Thus, the rate of change of $\bar{u}$ along the depth of the layer represents the hydraulic gradient. The velocity with which the excess pore water flows at the depth z is given by Darcy's law.

$$v = ki = \frac{k}{\gamma_w} \frac{\partial u}{\partial z} \longrightarrow ③ \quad \{\because ②\}$$

The rate of chang of velocity along the depth of the layer is then given by

$$\frac{\partial v}{\partial z} = \frac{k}{\gamma_w} \frac{\partial^2 \bar{u}}{\partial z^2} \longrightarrow ④$$

consider a small soil element of size dx, dz, and of width dy perpendicular to the xz plane. If v is the velocity of water at the entry into the elements, the velocity at the exit will be equal

to $v + \frac{\partial v}{\partial z} dz \longrightarrow$ (a)

The quantity of water entering the soil element $= v\,dx\,dy$

The quantity of water leaving the soil element $= \cancel{\left(v + \frac{\partial v}{\partial z} dz\right)} dy$

$$= \left(v + \frac{\partial v}{\partial z} dz\right) dx\,dy \longrightarrow (b)$$

Hence the net quantity of water dq squeezed out of the soil element per unit time is give by

$$\Delta q = \frac{\partial v}{\partial z}\, dx\,dy\,dz \longrightarrow (5) \quad \{\because (b) - (a)\}$$

The decrease in the volume of soil is equal to the volume of water squeezed out.

However from the eq $\Delta V = -m_v V_0 \Delta\sigma' \longrightarrow (6)$

where V_0 = volume of soil element at time $t_0 = dx\,dy\,dz$

$\therefore$ change of volume per unit time is given by

$$\frac{\partial(\Delta V)}{\partial t} = -m_v\, dx\,dy\,dz\, \frac{\partial(\Delta\sigma')}{\partial t} \longrightarrow (7)$$

Equating (5) and (7), we get

$$\frac{\partial v}{\partial z} = -m_v \frac{\partial(\Delta\sigma')}{\partial t} \longrightarrow (8)$$

Now $\Delta\sigma = \Delta\sigma' + \bar{u}$, where $\Delta\sigma$ is constant

$$\therefore \frac{\partial(\Delta\sigma')}{\partial t} = -\frac{\partial\bar{u}}{\partial t} \longrightarrow (9)$$

Hence, from (8) and (9) $\frac{\partial v}{\partial z} = m_v \frac{\partial\bar{u}}{\partial t} \longrightarrow (10)$

combining equations (4) and (10), we get

$$\frac{\partial\bar{u}}{\partial t} = \frac{k}{m_v \gamma_w} \frac{\partial^2 \bar{u}}{\partial z^2} \longrightarrow (11)$$

$$\frac{\partial\bar{u}}{\partial t} = C_v \frac{\partial^2 \bar{u}}{\partial z^2} \longrightarrow (12)$$

Where C_v = coefficient of consolidation $= \frac{k}{m_v \gamma_w}$

$$= \frac{k(1+e_0)}{a_v \gamma_w}$$

The eqn (12) is the basic differential equation consolidation which relates the rate of change of excess hydrostatic pressure to the rate of expulsion of excess pore water from a unit volume of soil during the same time interval.

The term coefficient of consolidation C_v used in the equation is adopted to indicate the combined effects of permeability and compressibility of soil on the rate of volume change. The units of C_v are cm^2/sec.

(1)

UNIT - VIII

13

Shear Strength

13.1. INTRODUCTION

The shear strength of a soil is its maximum resistance to shear stresses just before the failure. Soils are seldom subjected to direct shear. However, the shear stresses develop when the soil is subjected to direct compression. Although shear stresses may also develop when the soil is subjected to direct tension, but these shear stresses are not relevant, as the soil in this case fails in tension and does not fail in shear. In field, soils are seldom subjected to tension, as it causes opening of the cracks and fissures. These cracks are not only undesirable, but are also detrimental to the stability of the soil masses. Thus, the shear failure of a soil mass occurs when the shear stresses induced due to the applied compressive loads exceed the shear strength of the soil. It may be noted that the failure in soil occurs by relative movements of the particles and not by breaking of the particles.

Shear strength is the principal engineering property which controls the stability of a soil mass under loads. It governs the bearing capacity of soils, the stability of slopes in soils, the earth pressure against retaining structures and many other problems, as explained in later chapters. All the problems of soil engineering are related in one way or the other with the shear strength of the soil. Unfortunately, the shear strength is one of the most complex engineering properties of the soil. The current research is giving new concepts and theories. This chapter presents the basic concepts and the accepted theories of the shear strength.

13.2. STRESS-SYSTEM WITH PRINCIPAL PLANES PARALLEL TO THE COORDINATE AXES

In general, a soil mass is subjected to a three—dimensional stress system. However, in many soil engineering problems, the stresses in the third direction are not relevant and the stress system is simplified as two-dimensional. The plane strain conditions are generally assumed, in which the strain in the third (longitudinal) direction is zero. Such conditions exist, for example, under a strip footing of a long retaining wall.

At every point in a stressed body, there are three planes on which the shear stresses are zero. These planes are known as *principal planes*. The plane with the maximum compressive stress (σ_1) is called the major principal plane, and that with the minimum compressive (σ_3) as the minor principal plane. The third principal plane is subjected to a stress which has the value intermediate between σ_1 and σ_3, and is known as the intermediate principal plane. Generally, the stresses on a plane perpendicular to the intermediate principal plane are required in the analysis. Therefore, the stresses on the intermediate principal plane are not much relevant. Only the major principal stress (σ_1) and the minor principal stress (σ_3) are generally important.

In solid mechanics, the tensile stresses are taken as positive. In soil engineering problems, tensile stresses rarely occur. To avoid many negative signs, compressive stresses are taken as positive and the tensile stresses as negative in soil engineering.

Fig. 13.1 shows a plane which is perpendicular to the intermediate principal plane. The major and minor principal stresses act on this plane. The major principal plane is horizontal and the minor principal plane is

$$\sin 2\theta_p = \pm \frac{\tau_{xy}}{\sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2}}$$

$$\cos 2\theta_p = \pm \frac{(\sigma_y - \sigma_x)/2}{\sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2}}$$

Substituting these values of $\sin 2\theta_p$ and the $\cos 2\theta_p$ in Eq. 13.3,

$$\sigma = \frac{\sigma_x + \sigma_y}{2} \pm \left(\frac{\sigma_y - \sigma_x}{2}\right) \times \frac{(\sigma_y - \sigma_x)/2}{\sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2}} \pm \frac{\tau_{xy} \times \tau_{xy}}{\sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2}}$$

or
$$\sigma = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

Therefore, the two principal stresses are as under.

Major principal stress,
$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2} \qquad ...(13.7)$$

The point U gives the major principal stress (σ_1).

Minor principal stress,
$$\sigma_3 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2} \qquad ...(13.8)$$

The point V gives the minor principal stress (σ_3)

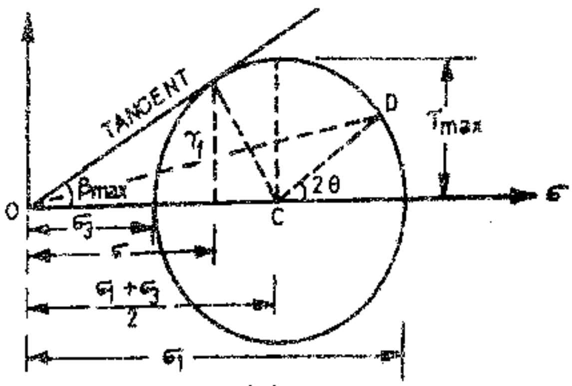
Also, because $\tan 2\theta_p = \tan(2\theta_p + 180°)$, the second principal plane is indicated by the line CV.

13.6. IMPORTANT CHARACTERISTICS OF MOHR'S CIRCLE

The following important characteristics of Mohr's circle should be carefully noted, as these are required for further study.

(1) The maximum shear stress τ_{max} is numerically equal to $(\sigma_1 - \sigma_3)/2$ and it occurs on a plane inclined at 45° to the principal planes (Fig. 13.5).

(2) Point D on the Mohr circle represents the stresses (σ, τ) on a plane make an angle θ with the major principal plane.



(c)

Fig. 13.5. Characteristics of Mohr's Circle.

The resultant stress on that plane is equal to $\sqrt{\sigma^2 + \tau^2}$ and its angle of obliquity with the normal of the plane is equal to angle β, given by

$$\beta = \tan^{-1}(\tau/\sigma) \qquad \ldots(13.9)$$

(3) The maximum angle of obliquity β_{max} is obtained by drawing a tangent to the circle from the origin O.

$$\beta_{max} = \sin^{-1}\frac{(\sigma_1 - \sigma_3)/2}{(\sigma_1 + \sigma_2)/2} = \sin^{-1}\left(\frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}\right) \qquad \ldots(13.9)$$

(4) The shear stress τ_f on the plane of the maximum obliquity is less than the maximum shear stress τ_{max}.

(5) Shear stresses on planes at right angles to each other are numerically equal but are of opposite signs, as shown in Fig. 13.4 (*c*).

(6) As the Mohr circle is symmetrical about σ-axis, it is usual practice to draw only the top half circle for convenience.

(7) There is no need to be rigid about sign convention for plotting the shear stresses in Mohr's circle. These can be plotted either upward or downward. Although the sign convention is required for locating the orientation of the planes, the numerical results are not affected.

13.7. MOHR-COULOMB THEORY

The soil is a particulate material. The shear failure occurs in soils by slippage of particles due to shear stresses. The failure is essentially by shear, but shear stresses at failure depend upon the normal stresses on the potential failure plane. According to Mohr, the failure is caused by a critical combination of the normal and shear stresses.

The soil fails when the shear stress (τ_f) on the failure plane at failure is a unique function of the normal stress (σ) acting on that plane.

$$\tau_f = f(\sigma)$$

Since the shear stress on the failure plane at failure is defined as the shear strength (s), the above equation can be written as

$$s = f(\sigma) \qquad \ldots(13.11)$$

The Mohr theory is concerned with the shear stress at failure plane at failure. A plot can be made between the shear stress τ and the normal stress σ at failure. The curve defined by Eq. 13.11 is known as the Mohr envelope [Fig. 13.6 (*a*)]. There is a unique failure envelope for each material.

Failure of the material occurs when the Mohr circle of the stresses touches the Mohr envelope. As discussed in the preceding sections, the Mohr circle represents all possible combinations of shear and normal stresses at the stressed point. At the point of contact (D) of the failure envelope and the Mohr circle, the critical combination of shear and normal stresses is reached and the failure occurs. The plane indicated by the line PD is, therefore, the failure plane. Any Mohr's circle which does not cross the failure envelope and

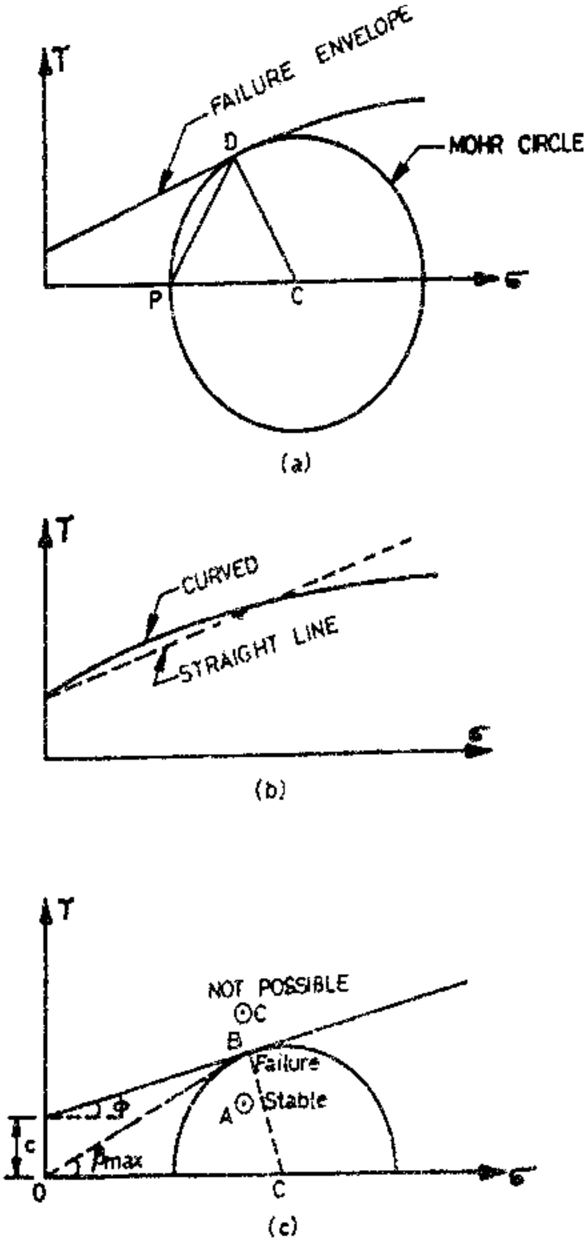


Fig. 13.6. Failure Envelopes.

lies below the envelope represents a (non-failure) stable condition. The Mohr circle cannot cross the Mohr envelope, as the failure would have already occurred as soon as the Mohr circle touched the envelope.

The shear strength (s) of a soil at a point on a particular plane was expressed by Coulomb as a linear function of the normal stress on that plane, as

$$s = c + \sigma \tan \phi \qquad \text{...(13.12)}$$

In other words, the Mohr envelope is replaced by a straight line by Coulomb as shown in Fig. 13.6 (*b*).

In Eq. 13.12, c is equal to the intercept on τ-axis and ϕ is the angle which the envelope makes with σ-axis [Fig. 13.6 (*c*)]. The component c of the shear strength is known as *cohesion*. Cohesion holds the particles of the soil together in a soil mass, and is independent of the normal stress. The angle ϕ is called the *angle of internal friction*. It represents the frictional resistance between the particles, which is directly proportional to the normal stress.

As mentioned before, the failure occurs when the stresses are such that the Mohr circle just touches the failure envelope, as shown by point B in Fig. 13.6 (*c*). In other words, shear failure occurs if the stresses σ and τ on the failure plane plot as point B. If the stresses plot as point A below the failure envelope, it represents a stable, non-failure condition. On the other hand, a state of stress represented by point C above the failure envelope is not possible. It may be noted that a material fails along a plane when the critical combination of the stresses σ and τ gives the resultant with a maximum obliquity (β_{max}), in which case the resultant just touches the Mohr circle.

13.8. REVISED MOHR-COULOMB EQUATION

Later research showed that the parameters c and ϕ in Eq. 13.12 are not necessarily fundamental properties of the soil as was originally assumed by Coulomb. These parameters depend upon a number of factors, such as the water content, drainage conditions, conditions of testing. The current practice is to consider c and ϕ as mathematical parameters which represent the failure conditions for a particular soil under given conditions. That is the reason why c and ϕ are now called cohesion intercept and the angle of shearing resistance. These indicate the intercept and the slope of the failure envelope, respectively.

Terzaghi established that the normal stresses which control the shear strength of a soil are the effective stresses and not the total stresses. In terms of effective stresses, Eq. 13.12 is written as

$$s = c' + \bar{\sigma} \tan \phi' \qquad \text{...(13.13)}$$

where c' and ϕ' are the cohesion intercept and the angle of shearing resistance in terms of the effective stresses.

Eq. 13.13 is known as the *Revised Mohr—Coulomb* equation for the shear strength of the soil. The equation has replaced the original equation (Eq. 13.12). It is one of the most important equations of soil engineering.

The Mohr—Coulomb theory shows a reasonably good agreement with the observed failures in the field and in the laboratory. The theory is ideally suited for studying the behaviour of soils at failure. The theory is used for estimation of the shear strength of soils. However, even this theory is not perfect. It has the following main limitations :

(1) It neglects the effect of the intermediate principal stress (σ_2).

(2) It approximates the curved failure envelope by a straight line, which may not give correct results.

(3) When the Mohr envelope is curved, the actual obliquity of the failure plane is slightly smaller than the maximum obliquity. Therefore, the angle of the failure plane, as found, is not correct.

(4) For some clayey soils, there is no fixed relationship between the normal and shear stresses on the plane of failure. The theory cannot be used for such soils.

13.9. DIFFERENT TYPES OF TESTS AND DRAINAGE CONDITIONS

The following tests are used to measure the shear strength of a soil.

(1) Direct shear test (2) Triaxial Compression test

(3) Unconfined Compression test (4) Shear Vane test.

The shear test must be conducted under appropriate drainage conditions that simulate the actual field problem. In shear tests, there are two stages :

(1) Consolidation stage in which the normal stress (or confining pressure) is applied to the specimen and it is allowed to consolidate.

(2) Shear stage in which the shear stress (or deviator stress) is applied to the specimen to shear it.

Depending upon the drainage conditions, there are three types of tests as explained below :

(1) Unconsolidated—Undrained Condition. In this type of test, no drainage is permitted during the consolidation stage. The drainage is also not permitted in the shear stage.

As no time is allowed for consolidation or dissipation of excess pore water pressure, the test can be conducted quickly in a few minutes. The test is known as unconsolidated—undrained test (*UU* test) or quick test (*Q*-test).

(2) Consolidated—Undrained Condition. In a consolidated---undrained test, the specimen is allowed to consolidate in the first stage. The drainage is permitted until the consolidation is complete.

In the second stage when the specimen is sheared, no drainage is permitted. The test is known as consolidated—undrained test (*CU* test) It is also called a *R*-test, as the alphabet *R* falls between the alphabet *Q* used for quick test, and the alphabet *S* used for slow test.

The pore water pressure can be measured in the second stage if the facilities for its measurement are available. In that case, the test is known as $\overline{CU}$ test.

(3) Consolidated---Drained Condition. In a consolidated—drained test, the drainage of the specimen is permitted in both the stages. The sample is allowed to consolidate in the first stage. When the consolidation is complete, it is sheared at a very slow rate to ensure that fully drained conditions exist and the excess pore water is zero.

The test is known as a consolidated—drained test (*CD* test) or drained test. It is also known as the slow test (*S*—test).

13.10. MODE OF APPLICATION OF SHEAR FORCE

The shear force in a shear test is applied either by increasing the shear displacement at a given rate or by increasing the shearing force at a given rate. Accordingly, the shear tests are either strain—controlled or stress-controlled.

(1) Strain controlled tests. In a strain-controlled test, the test is conducted in such a way that the shearing strain increases at a given rate. Generally, the rate of increase of the shearing strain is kept constant, and the specimen is sheared at a uniform strain rate.

The shear force acting on the specimen is measured indirectly using a proving ring. The rate of shearing strain is controlled manually or by a gear system attached to an electric motor.

Most of the shear tests are conducted as strain—controlled. The stress—strain characteristic are easily obtained in these tests, as the shape of the stress—strain curve beyond the peak point can be observed only in a strain—controlled test. A strain—controlled test is easier to perform than a stress- controlled test.

(2) Stress—Controlled tests. In a stress—controlled test, the shear force is increased at a given rate. Usually, the rate of increase of the shear force is maintained constant. The shear load is increased such that the shear stresses increase at a uniform rate. The resulting shear displacements are obtained by means of a dial gauge.

Stress—controlled tests are preferred for conducting shear tests at a very low rate, because an applied load can easily be kept constant for any given period of time. Further, the loads can be conveniently applied and removed. The stress-controlled test represents the field conditions more closely.

13.11 DIRECT SHEAR TEST

(*a*) Apparatus. A direct shear test is conducted on a soil specimen in a shear box which is split into two halves along a horizontal plane at its middle (Fig. 13.7). The shear box is made of brass or gunmetal. It is

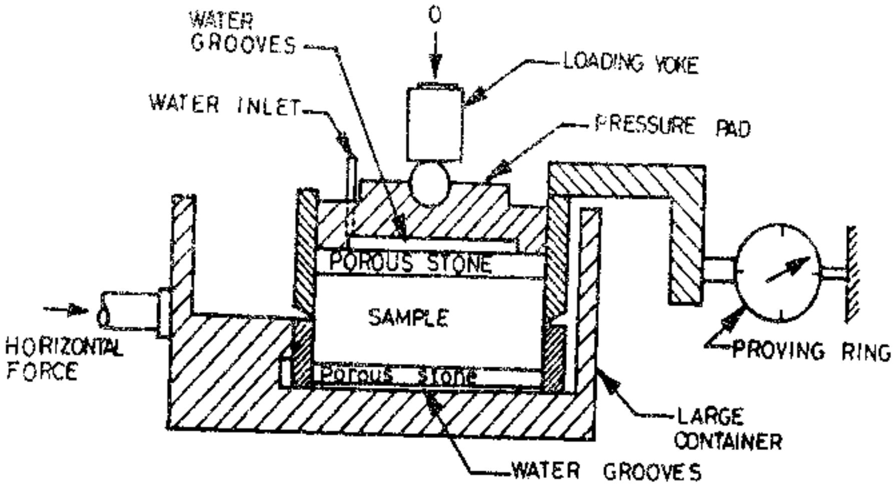


Fig. 13.7. Direct Shear Test.

either square or circular in plan. A square box of size 60 × 60 × 50mm is commonly used. The box is divided horizontally such that the dividing plane passes through the centre. The two halves of the box are held together by locking pins. Suitable spacing screws to separate the two halves are also provided. The spacing screws are fixed to the upper half and they butt against the top of the lower half.

The box is provided with the gripper or the grid plates which are toothed and fitted inside it. The gripper plates are plain (without perforations) for undrained tests and perforated for drained tests. Porous stones are placed at the top and the bottom of the specimen in drained tests. A pressure pad of brass or gun metal is fitted into the box at its top to transmit the normal load to the sample. The normal load from the loading yoke is applied on the top of the specimen through a steel ball bearing upon the pressure pad.

The lower half of the box is fixed to the base plate which is rigidly held in position in a large container. The large container is supported on rollers (rollers not shown). The container can be pushed forward at a constant rate by a geared jack which works as a strain-controlled device. The jack may be operated manually or by an electric motor.

A loading frame is used to support the large container. It has the arrangement of a loading yoke and a lever system for applying the normal load.

A proving ring is fitted to the upper half of the box to measure the shear force.The proving ring butts against a fixed support. As the box moves, the proving ring records the shear force. The shear displacement is measured with a dial gauge fitted to the container. Another dial gauge is fitted to the top of the pressure pad to measure the change in the thickness of the specimen.

(*b*) Test. A soil specimen of size 60 × 60 × 25 mm is taken. It may be either an undisturbed sample or made from compacted and remoulded soil. The specimen may be prepared directly in the box and compacted. The base plate is attached to the lower half of the box. A porous stone is placed in the box. For undrained tests, a plain grid is kept on the porous stone, keeping its segregations at right angles to the direction of shear. For drained tests, perforated grids are used instead of plain grids. The mass of the base plate, porous stone and grid is taken. The specimen if made separately is transferred to the box and its mass taken.

The upper grid, porous stone and the pressure pad are placed on the specimen. The box is placed inside the large container and mounted on the loading frame. The upper half of the box is brought in contact with the proving ring. The loading yoke is mounted on the steel ball placed on the pressure pad. The dial gauge is fitted to the container to give the shear displacement. The other dial gauge is mounted on the loading yoke to record the vertical movement.

The locking pins are removed and the upper half box is slightly raised with the help of spacing screws. The space between the two halves is adjusted, depending upon the maximum particle size. The space should be such that the top half of the box does not ride on soil grains which come between the edges.

The normal load is applied to give a normal stress of 25 kN/m^2. Shear load is then applied at a constant rate of strain. For undrained tests, the rate is generally between 1.0 mm to 2.00 mm per minute. For drained tests, the strain rate depends upon the type of soil. For sandy soils, it may be taken as 0.2 mm/minute; whereas for clayey soils, it is generally between 0.005 to 0.02 mm/min. The sample shears along the horizontal plane between the two halves. The readings of the proving-ring and the dial gauges are taken every 30 seconds. The test is continued till the specimen fails. The failure is indicated when the proving ring dial gauge begins to recede after having reached the maximum. For the soils which do not give a peak point, the failure is assumed to have occurred when a shearing strain of 20% is reached. At the end of the test, the specimen is removed from the box and its water content found.

The test is repeated under the normal stress of 50, 100, 200 and 400 kN/m^2. The range of the normal stress should cover the range of loading in the field problem for which the shear parameters are required. The shear stress at any stage during shear is equal to the shear force indicated by the proving ring divided by the area of the specimen. A plot can be made between the shear stress and the shear strain. The shear strain is equal to the shear displacement (ΔH) divided by the length of the specimen (L). The shear stress is obtained from the shear load indicated by the proving ring and the cross-sectional area.

Direct shear tests can be conducted for any one of the three drainage conditions. For *U-U* test, plain grids are used and the sample is sheared rapidly. For *CU* test, perforated grids are used. The sample is consolidated under the normal load and after the completion of consolidation, it is sheared rapidly in about 5—10 minutes. In a *CD* test, the sample is consolidated under the normal load and then sheared slowly so that excess pore water pressure is dissipated. A *CD* test may take a few hours for cohesionless soils. For cohesive soils, it may take 2 to 5 days.

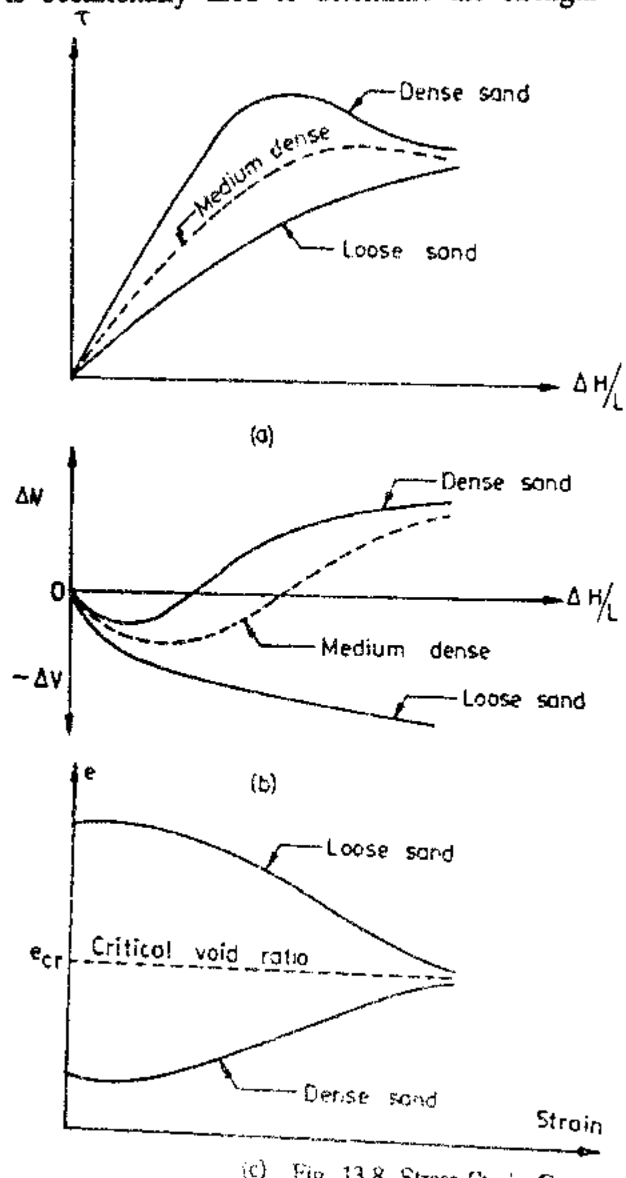
The direct shear test is generally conducted on cohesionless soils as *CD* test. It is convenient to perform and it gives good results for the strength parameters. It is occasionally used to determine the strength parameters of silt and clay under unconsolidated—undrained and consolidated drained conditions, but it does not offer the flexibility of a triaxial compression test, as explained later.

13.12. PRESENTATION OF RESULTS OF DIRECT SHEAR TEST

(*a*) **Stress-Strain Curve.** A stress-strain curve is a plot between the shear stress τ and the shear displacement ($\Delta H/L$) [Fig. 13.8 (*a*)]. In case of dense sand (and also over-consolidated clays), the shear stress attains a *peak value* at a small strain. With further increase in strain, the shear stress decreases slightly and becomes more or less constant, known as ultimate stress. In case of loose sands (and normally consolidated clays), the shear stress increases gradually and finally attains a constant value, known as the *ultimate stress* or residual strength. It has been observed that the ultimate shear stress attained by both dense and loose sands tested under similar conditions is approximately the same. The figure also shows the stress-strain curve of a medium dense sand.

Generally, the failure strain is 2 to 4% for dense sand and 12 to 16% for loose sand.

Fig. 13.8 (*b*) shows the volume changes with an increase in shear strain for *CD* tests. Since the cross-sectional area of the specimen remains unchanged, the volume change is proportional to the change in thickness measured by the dial gauge. In case of dense sands (and over-consolidated clays), the volume first decreases slightly,



(c) Fig. 13.8. Stress-Strain Curves.

but it increases with further increase in strain. In the case of loose sands (and normally consolidated clays), the volume decreases with an increase in shear strain. The figure also shows the curve for medium dense sand.

It may be observed that the void ratio of an initial loose sand decreases with an increase in shear strain, whereas that for the initially dense sand increases with an increase in strain [Fig. 13.8 (c)]. The void ratio at which there is no change in it with an increase in strain is known as the *critical void ratio.* If the sand initially is at the critical void ratio, there would be practically no change in volume with an increase in shear strain.

(*b*) **Failure Envelope.** For obtaining a failure envelope, a number of identical specimens are tested under different normal stresses. The shear stress required to cause failure is determined for each normal stress. The failure envelope is obtained by plotting the points corresponding to shear strength at different normal stresses and joining them by a straight line [Fig. 13.9 (*a*)]. The inclination of the failure envelope to

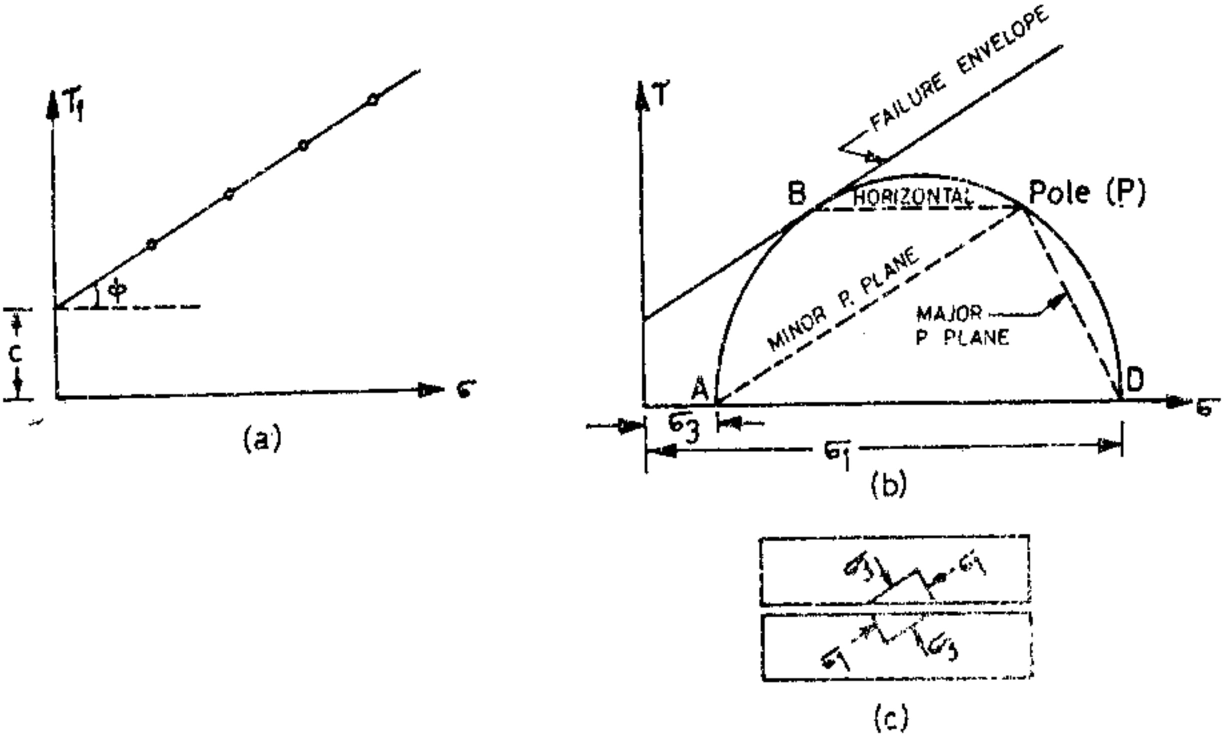


Fig. 13.9. Failure Envelope.

the horizontal gives the angle of shearing resistance ϕ and its intercept on the vertical axis is equal to the cohesion intercept c.

For dense sands, the failure envelope can be drawn either for peak stress or for ultimate stress. The values of the parameters ϕ and c for the two envelopes will be different. For loose sands, the failure envelope is drawn for ultimate stress, which is usually taken as the shear stress at 20% shear strain.

(c) **Mohr-Circle.** In a direct shear test, the stresses on planes other than the horizontal plane are not known. It is, therefore, not possible to draw Mohr stress circle at different shear loads. However, the Mohr circle can be drawn at the failure condition assuming that the failure plane is horizontal.

In Fig. 13.9 (*b*), the point *B* represents the failure condition for a particular normal stress. The Mohr circle at failure is drawn such that it is tangential to the failure envelope at *B*. The horizontal line *BP* gives the direction of the failure plane. The point *P* is the pole. The lines *PD* and *PA* give the directions of the major and minor principal planes, respectively. The principal planes are also shown in Fig. 13.9 (*c*).

Merits and Demerits of Direct Shear Test

The direct shear test has the following merits and demerits as compared to the triaxial compression test (described in the following section).

Merits.

(1) The sample preparation is easy. The test is simple and convenient.

(2) As the thickness of the sample is relatively small, the drainage is quick and the pore pressure dissipates very rapidly. Consequently, the consolidated-drained and the consolidated- undrained tests take relatively small period.

(3) It is ideally suited for conducting drained tests on cohesionless soils.

(4) The apparatus is relatively cheap.

Demerits.

(1) The stress conditions are known only at failure. The conditions prior to failure are indeterminate and, therefore, the Mohr circle cannot be drawn.

(2) The stress distribution on the failure plane (horizontal plane) is not uniform. The stresses are more at the edges and lead to the progressive failure, like tearing of a paper. Consequently, the full strength of the soil is not mobilised simultaneously on the entire failure plane.

(3) The area under shear gradually decreases as the test progresses. But the corrected area cannot be determined and, therefore, the original area is taken for the computation of stresses.

(4) The orientation of the failure plane is fixed. This plane may not be the weakest plane.

(5) Control on the drainage conditions is very difficult. Consequently, only drained tests can be conducted on highly permeable soils.

(6) The measurement of pore water pressure is not possible.

(7) The side walls of the shear box cause lateral restraint on the specimen and do not allow it to deform laterally.

13.13. DIFFERENT TYPES OF SOILS

On the basis of shear strength, soils can be divided into three types.

(1) Cohesionless soils. (2) Purely cohesive soils and

(3) Cohesive-frictional soils.

1. Cohesionless soils. These are the soils which do not have cohesion *i.e.*, $c' = 0$. These soils derive the shear strength from the intergranular friction. These soils are also called *frictional soils.* For example, sands and gravels.

2. Purely cohesive soils. These are the soils which exhibit cohesion but the angle of shearing resistance $\phi = 0$. For example, saturated clays and silts under undrained conditions. These soils are also called $\phi_u = 0$ soils.

3. Cohesive-frictional soils. These are composite soils having both c' and ϕ'. These are also called c—ϕ soils. For example, clayey sand, silty sand, sandy clay, etc.

[**Note.** Sometimes, cohesive-frictional soils are also called cohesive soils. Thus any soil having a value of c' is called a cohesive soil.]

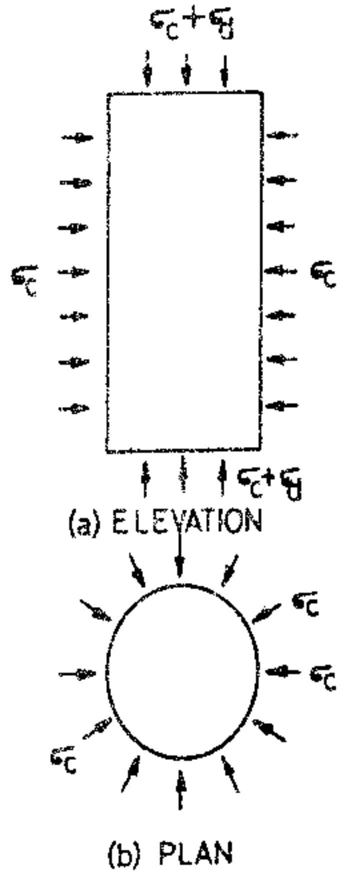


Fig. 13.10.

13.14. TRIAXIAL COMPRESSION TEST APPARATUS

The triaxial compression test, or simply triaxial test, is used for the determination of shear characteristics of all types of soils under different drainage conditions. In this test, a cylindrical specimen is stressed under conditions of axial symmetry, as shown in Fg. 13.10. In the first stage of the test, the specimen is subjected to an all round confining pressure (σ_c) on the sides and at the top and the bottom. This stage is known as the consolidation stage.

In the second stage of the test, called the shearing stage, an additional axial stress, known as the deviator stress (σ_d), is applied on the top of the specimen through a ram. Thus, the total stress in the axial direction at the time of shearing is equal to $(\sigma_c + \sigma_d)$. It may be noted that when the axial stress is increased, the shear stresses develop on inclined planes due to compressive stresses on the top.

The vertical sides of the specimen are principal planes, as there are no shear stresses on the sides. The confining pressure σ_c is equal to the minor principal stress (σ_3). The top and bottom planes are the major principal planes. The total axial stress which is equal to the sum of the confining pressure and the deviator stress, is the major principal stress (σ_1). Because of axial symmetry, the intermediate principal stress (σ_2) is also equal to the confining pressure (σ_c).

[**Note.** The above interpretation of the stress conditions in the triaxial test is not strictly correct according to the theory of elasticity. In the case of cylindrical specimens, the three principal stresses are the axial, radial and the circumferential stresses. The state of stress is statically indeterminate throughout the specimen. For convenience, in the triaxial test, the circumferential stress is taken equal to the radial stress and the principal stresses σ_2 and σ_3 are assumed to be equal].

The main features of a triaxial test apparatus are shown in Fig. 13.11. It consists of a circular base that has a central pedestal. The pedestal has one or two holes which are used for the drainage of the specimen in a drained test or for the pore pressure measurement in an undrained test. A triaxial cell is fitted to the top of the base plate with the help of 3 wing nuts (not shown in the figure) after the specimen has been placed on the pedestal. The triaxial cell is a perspex cylinder which is permanently fixed to the top cap and the bottom brass collar. There are three tie rods which support the cell. The top cap is a bronze casting with its central boss forming a bush through which a stainless steel ram can slide. The ram is so designed that it has minimum of friction and at the same time does not permit any leakage. There is an air-release valve in the top cap which is kept open when the cell is filled with water (or glycerine) for applying the confining pressure. An oil valve is also provided in the top cap to fill light machine oil in the cell to reduce the leakage of water past the ram in long duration tests. The apparatus is mounted on a loading frame. The deviator stress is applied to the specimen from a strain-controlled loading machine. The loading system consists of either a screw jack operated by an electric motor and gear box or a hydraulic ram operated by a pump.

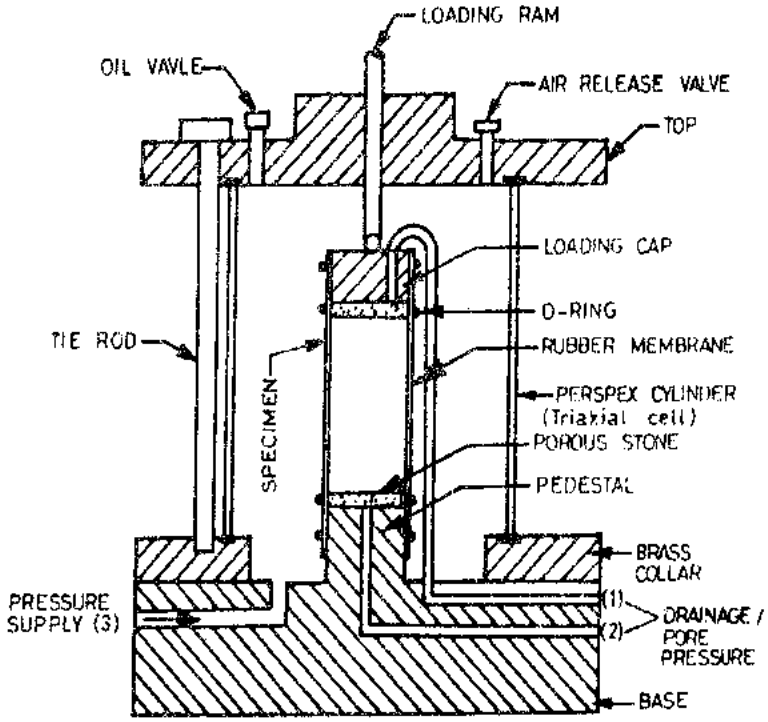


Fig. 13.11. Triaxial Tes Apparatus.

The triaxial test apparatus has the following special attachments.

1. Mercury Control System. The cell pressure is a triaxial test in maintained constant with a self-compensating mercury control system, developed by Bishop and Henkel. It consists of two limbs of a water-mercury manometer (Fig. 13.12). The pressure in the water of the triaxial cell develops due to the difference in levels of the mercury in the two pots. The water pressure at the centre of the specimen in the triaxial cell, at a height of h_3 above the datum, can be calculated using the theory of manometers. As the mercury surface in the upper pot is open to atmosphere, the (gauge) pressure there is zero. From the manometer equation,

$$0 + \gamma_m h_1 - \gamma_m h_2 - (h_3 - h_2)\,\gamma_w = \sigma_c$$

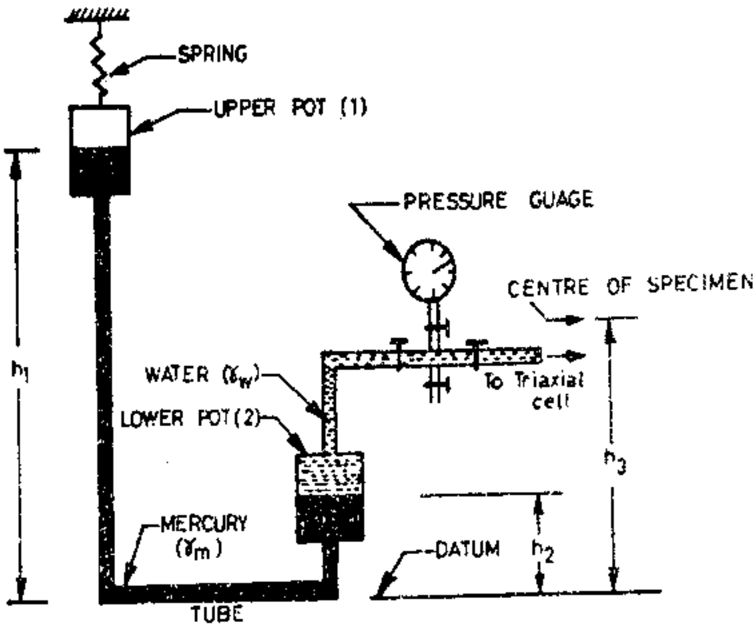


Fig. 13.12. Mercury Control System.

where σ_c is the cell pressure at the centre of the specimen,

γ_w is the unit weight of water, and

γ_m is the unit weight of mercury.

The above equation can be simplified as

$$\sigma_c = \gamma_m (h_1 - h_2) + (h_2 - h_3) \gamma_w \qquad ...(13.14)$$

The upper pot is supported by a spring. When the volume of the specimen decreases due to consolidation or when the water leaks past the ram, water flows from the lower pot to the cell and the mercury level in the lower pot rises by a small amount Δh. The mercury level in the upper pot would also fall by the same amount if the two pots are of the same cross-sectional area. However, the difference of mercury levels in the two pots is maintained constant by the spring. The stiffness (k) of the spring is selected such that it reduces in length and causes a rise of the upper pot as soon as its weight decreases due to flow of mercury. The stiffness of the spring is given by

$$k = A\ \gamma_m \left[\frac{1}{2 - (\gamma_w/\gamma_m)} \right] - W \qquad ...(13.15)$$

where A = cross-sectional area of the mercury pot,

and W = weight of unit length of the tube filled with mercury which is also lifted above the floor.

2. Pore water Pressure Measurement Device. The pore water pressure in the triaxial specimen is measured by attaching it to the device shown in Fig. 13.13. It consists of a null indicator in which no- flow condition is maintained. For accurate measurement, *no- flow condition is essential because* the flow of water from the sample to the gauge would modify the actual magnitude of the pore water pressure. Further, the flow of water leads to a time lag in the attainment of a steady state in samples of cohesive soils because of low permeability.

The null indicator is essentially a U-tube partly filled with mercury. One limb of the null indicator is connected to the specimen in the triaxial cell and the other limb is connected to a pressure gauge. A control cylinder, which is filled with water, is attached to the system. The water can be displaced by a screw-controlled piston in the control cylinder. The whole system is filled with deaired water. The tubes connecting the specimen and the null-indicator should be such that these undergo negligible volume changes under pressure and are free from leakage.

Any change in the pore-water pressure in the specimen tends to cause a movement of the mercury level in the null-indicator. However, the no-flow condition is maintained by making a corresponding change in the

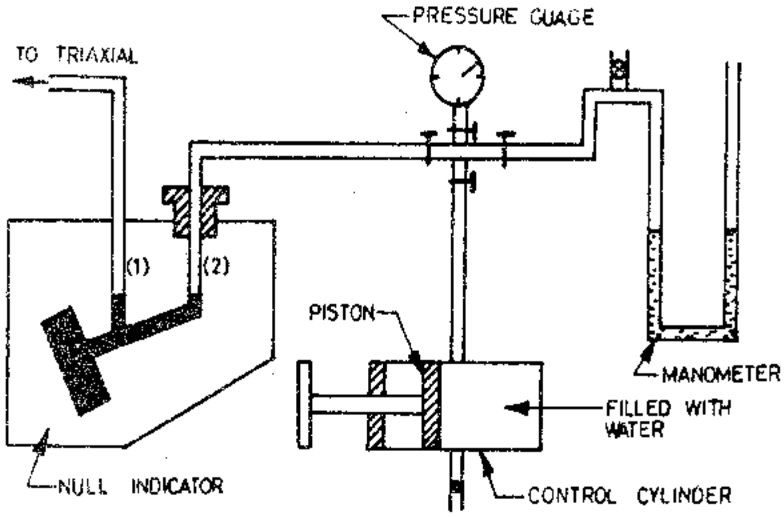


Fig. 13.13. Pore Water Pressure Measurement Device.

other limbs by means of the control cylinder. Thus the mercury levels in the two limbs remain constant. The pressure applied by the control cylinder is recorded by pressure gauge or the manometer.

If the specimen is partially saturated, a special fine, porous ceramic disc is placed below the sample in the triaxial cell. The ceramic disc permits only pore water to flow, provided the difference between the pore air pressure and pore water pressure is below a certain value, known as *the air-entry value* of the ceramic disc. Under undrained conditions, the ceramic disc will remain fully saturated, provided the air-entry value is high. It may be mentioned that if the required ceramic disc is not used and instead the usual coarse, porous disc is used, the device would measure air pressure and not water pressure in a partially saturated soil.

In modern equipment, sometimes the pore water pressure is measured by means of a transducer and not by conventional null indicator.

3. Volume Changes Measurement. Volume changes in a drained test and during consolidation stage of a consolidated undrained test are measured by means of a burette connected to the specimen in the triaxial cell. For accurate measurements, the water level in the burette should be approximately at the level of the centre of the specimen (Fig. 13.14).

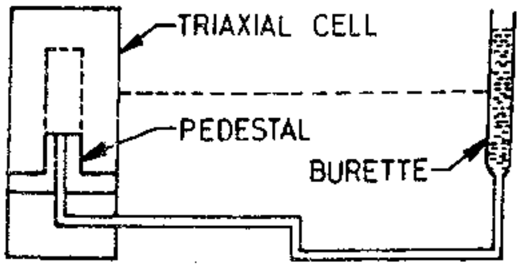


Fig. 13.14. Volume Change Measurement.

During consolidation stage, the volume of the specimen decreases and the water level in the burette rises. The change in the volume of the specimen is equal to the volume of the water increased in the burette. During shearing of specimens of dense sand when the volume of the sample increases, the water flows from the burette to the specimen. The increase in volume of the specimen is equal to the volume of water decreased in the burette.

13.15. TRIAXIAL TESTS ON COHESIVE SOILS

The following procedure is used for conducting the triaxial tests an cohesive soils.

(a) **Consolidated-undrained test.** A deaired, coarse porous disc or stone is placed on the top of the pedestal in the triaxial test apparatus. A filter paper disc is kept over the porous stone. The specimen of the cohesive soil is then placed over the filter paper disc. The usual size of the specimen is about 37.5 mm diameter and 75.0 mm height. A porous stone is also placed on the top of the specimen. Deaired vertical filter strips are placed at regular spacing around the entire periphery such that these touch both the porous stones. The sample in then enclosed in a rubber membrane, which is slid over the specimen with the help of a membrane stretcher. The membrane is sealed to the specimen with O-rings.

The triaxial cell is placed over the base and fixed to it by tightening the nuts. The cell is then filled with water by connecting it to the pressure supply. Some space in the top portion of the cell is filled by injecting oil through the oil valve. When excess oil begins to spill out through the air-vent valve, both the valves (oil valve and air-vent valve) are closed. Pressure is applied to the water filled in the cell by connecting it to the mercury-pot system. As soon as the pressure acts on the specimen, it starts consolidating. The specimen is connected to the burette through pressure connections for measurement of volume changes. The consolidation is complete when there is no more volume change.

When the consolidation is complete, the specimen is ready for being sheared. The drainage valve is closed. The pore water pressure measurement device is attached to the specimen through the pressure connections. The proving ring dial gauge is set to zero. Using the manual control provided in the loading frame, the ram is pushed into the cell but not allowed to touch the loading cap. The loading machine is then run at the selected speed. The proving ring records the force due to friction and the upward thrust acting on the ram. The machine is stopped, and with the manual control, the ram is pushed further into the cell bringing it in contact with the loading cap. The dial gauge for the measuring axial deformation of the specimen is set to zero.

The sample is sheared by applying the deviator stress by the loading machine. The proving ring readings are generally taken corresponding to axial strains of 1/3%, 2/3%, 1%, 2%, 3%, 4%, 5%, ...until failure or 20% axial strain.

Upon completion of the test, the loading is shut off. Using the manual control, all additional axial stress is removed. The cell pressure is then reduced to zero, and the cell is emptied. The triaxial cell is unscrewed and removed from the base. O-rings are taken out, and the membrane is removed. The specimen is then recovered after removing the loading cap and the top porous stone. The filter paper strips are peeled off. The post- shear mass and length are determined. The water content of the specimen is also found.

(*b*) **Unconsolidated Undrained test.** The procedure is similar to that for a consolidated-undrained test, with one basic difference that the specimen is not allowed to consolidate in the first stage. The drainage valve during the test is kept closed. However, the specimen can be connected to the pore-water pressure measurement device if required.

Shearing of the specimen is started just after the application of the cell pressure. The second stage is exactly the same as in the consolidated-undrained test described above.

(*c*) **Consolidated Drained test.** The procedure is similar to that for a consolidated-undrained test, with one basic difference that the specimen is sheared slowly in the second stage. After the consolidation of the specimen in the first stage, the drainage valve is not closed. It remains connected to the burette throughout the test. The volume changes during the shearing stage are measured with the help of the burette. As the permeability of cohesive soils is very low, it takes 4-5 days for the consolidated drained test.

13.16. TRIAXIAL TESTS ON COHESIONLESS SOILS

Triaxial tests on specimens of cohesionless soils can be conducted using the procedure as described for cohesive soils. As the samples of cohesionless soils cannot stand of their own, a special procedure is used for preparation of the sample as described below.

A metal former, which is a split mould of about 38.5 mm internal diameter, is used for the preparation of the sample (Fig. 13.15). A coarse porous stone is placed on the top of the pedestal of the triaxial base, and the pressure connection is attached to a burette (not shown). One end of a membrane is sealed to the pedestal by O-rings. The metal former is clamped to the base. The upper metal ring of the former is kept inside the top end of the rubber membrane and is held with the help of a clamp before placing the funnel and the rubber bung in position as shown in figure.

The membrane and the funnel are filled with deaired water. The cohesionless soil which is to be tested is saturated by mixing it with enough water in a beaker. The mixture is boiled to remove the entrapped air. The saturated soil is deposited in the funnel, with a stopper in position, in the required quantity. The glass rod is then removed and the sample builds up by a continuous rapid flow of saturated soil in the former. The funnel is then removed. The sample may be compacted if required. The surface of the sample is leveled and a porous stone is placed on its top. The loading cap is placed gently on the top porous stone. O-rings are fixed over the top of the rubber membrane.

A small negative pressure is applied to the sample by lowering the burette. The negative pressure gives rigidity to the sample and it can stand without any lateral support. For sample of 37.5 mm diameter, a negative pressure of 20 cm of water (or 2 kN/m^2) is sufficient. As soon as the negative pressure is applied, the consolidation of the sample occurs and it slightly shortens. The diameter of the upper porous stone should be slightly smaller than that of the specimen so that it can go inside when the sample shortens; otherwise, a neck is formed.

The split mould is then removed, and the diameter and the height of the sample are measured. The thickness of the membrane is deducted from the total diameter to get the net diameter of the sample. The cell is then placed over the base and clamped to the base. It is then filled with water.

The rest of the procedure is the same as for cohesive soils.

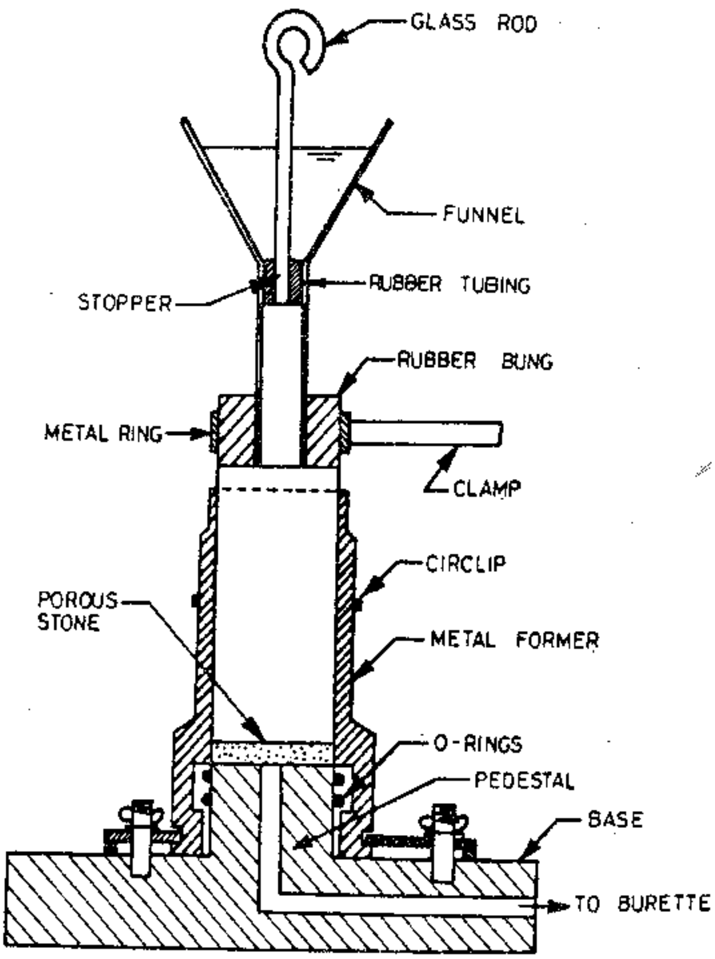


Fig. 13.15. Preparation of Sample of Cohesionless Soil.

13.17. MERITS AND DEMERITS OF TRIAXIAL TEST

The triaxial test has the following merits and demerits.

Merits.

(1) There is complete control over the drainage conditions. Tests can be easily conducted for all three types of drainage conditions.

(2) Pore pressure changes and the volumetric changes can be measured directly.

(3) The stress distribution on the failure plane is uniform.

(4) The specimen is free to fail on the weakest plane.

(5) The state of stress at all intermediate stages upto failure is known. The Mohr circle can be drawn at any stage of shear.

(6) The test is suitable for accurate research work. The apparatus is adaptable to special requirements such as extension test and tests for different stress paths.

Demerits.

(1) The apparatus is elaborate, costly and bulky.

(2) The drained test takes a longer period as compared with that in a direct shear test.

(3) The strain condition in the specimen are not uniform due to frictional restraint produced by the loading cap and the pedestal disc. This leads to the formation of the dead zones at each end of the specimen.

The non-uniform distribution of stresses can be largely eliminated by lubrication of end surfaces. However, non-uniform distribution of stresses has practically no effect on the measured strength if length/diameter ratio is equal to or more than 2.0.

(4) It is not possible to determine the cross-sectional area of the specimen accurately at large strains, as the assumption that the specimen remains cylindrical does not hold good.

(5) The test simulates only axis-symmetrical problems. In the field, the problem is generally 3-dimensional. A general test in which all the three stresses are varied would be more useful.

(6) The consolidation of the specimen in the test is isotropic; whereas in the field, the consolidation is generally anisotropic.

Despite the above-mentioned demerits, the triaxial test is extremely useful. It is the only reliable test for accurate determination of the shear characteristics of all types of soils and under all the drainage conditions.

13.18. COMPUTATION OF VARIOUS PARAMETERS

(a) Post-Consolidation Dimensions. In consolidated-drained and consolidated-undrained tests, the consolidation of the specimen takes place during the first stage. As the volume of the specimen decreases, its post-consolidation dimensions are different from the initial dimensions. The post consolidation dimensions can be determined approximately assuming that the sample remains cylindrical and it behaves isotropically. Let L_i, D_i, and V_i be the length, diameter and the volume of the specimen before consolidation. Let L_0, D_0 and V_0 be the corresponding quantities after consolidation.

Therefore, volumetric change, $\Delta V_i = V_i - V_0$

The volumetric change (ΔV_i) is measured with the help of burette.

Volumetric strain, $$\varepsilon_v = \frac{\Delta V_i}{V_i}$$

For isotropic consolidation, the volumetric strain is three times the linear strain (ε_l). Thus

$$\varepsilon_l = \varepsilon_v/3$$

Thus $$L_0 = L_i - \Delta L_i = L_i - L_i \times \varepsilon_l$$

or $$L_0 = L_i(1 - \varepsilon_l) = L_i\,(1 - \varepsilon_v/3) \qquad ...(13.16)$$

Likewise, $$D_0 = D_i(1 - \varepsilon_v/3)$$

The post consolidation diameter D_0 can also be computed after L_0 has been determined from the relation,

$$(\pi/4 \cdot D_0^2) \times L_0 = V_0$$

or $$D_0 = \sqrt{\frac{V_0}{(\pi/4) \times L_0}} \qquad ...(13.17)$$

(b) Cross-sectional Area During Shear Stage. As the sample is sheared, its length decreases and the diameter increases. The cross-sectional area A at any stage during shear can be determined assuming that the sample remains cylindrical in shape. Let ΔL_0 be the change in length and ΔV_0 be the change in volume. The volume of the specimen at any stage is given by $V_0 \pm \Delta V_0$.

Therefore, $$A\,(L_0 - \Delta L_0) = V_0 \pm \Delta V_0$$

or $$A = \frac{V_0 \pm \Delta V_0}{L_0 - \Delta L_0} = \frac{V_0\left(1 \pm \dfrac{\Delta V_0}{V_0}\right)}{L_0\left(1 - \dfrac{\Delta L_0}{L_0}\right)} \qquad ...(13.18)$$

Eq. 13.18 is the general equation which gives the cross-sectional area of the specimen.

The above equation can be written as

(PI) of the soil remains constant (Fig. 13.26). An approximate value of the undrained shear strength of a normally consolidated deposit can be obtained from Fig. 13.26, if the plasticity index has been determined. The relationship is expressed as (Skempton, 1957).

$$\frac{c_u}{\bar{\sigma}} = 0.11 + 0.0037\ PI$$

where c_u = undrained cohesion intercept,

$\bar{\sigma}$ = effective over-burden pressure

PI = plasticity index (%)

The value of the ratio ($c_u / \bar{\sigma}$) determined in a consolidated-undrained test on undisturbed samples is generally greater than actual value because of anisotropic consolidation in the field. The actual value is best determined by in-situ shear vane test, as explained later.

13.22. UNCONFINED COMPRESSION TEST

The unconfined compression test is a special form of a triaxial test in which the confining pressure is zero. The test can be conducted only on clayey soils which can stand without confinement. The test is generally performed on intact (non- fissured), saturated clay specimens. Although the test can be conducted in a triaxial test apparatus as a U–U test, it is more convenient to perform it in an unconfined compression testing machine. There are two types of machines, as described below.

(1) Machine with a spring. Fig. 13.27 shows the unconfined compression testing machine in which a loaded spring is used. It consists of two metal cones which are fixed on horizontal loading plates B and C supported on the vertical posts D. The upper loading plate B is fixed in position, whereas the lower plate C can slide on the vertical posts The soil specimen is placed between the two metal cones.

When the handle is turned, the plate A is lifted upward. As the plate A is attached to the plate C, the latter plate is also lifted. When the handle is turned slowly, at a speed of about half a turn per second, a compressive force acts on the specimen. Eventually, the specimen fails in shear. The compressive load is proportional to the extension of the spring.

The strain in the specimen is indicated on a chart fixed to the machine. As the lower plate C moves upward, the pen attached to this plate swings sideways. The lateral movement of the pen (in arc) is proportional to the strain in the specimen.

The chart plate is attached to the yoke Y. As the yoke moves upward when the handle is rotated, the chart plate moves upward. The pivot of the arm of the pen also moves upward with the lower plate. The vertical movement of the pen relative to the chart is equal to the extension of the spring and hence the compressive force. Thus the chart gives a plot between the deformation and the compressive force. Springs of different stiffnesses can be used depending upon the expected compressive strength of the specimen.

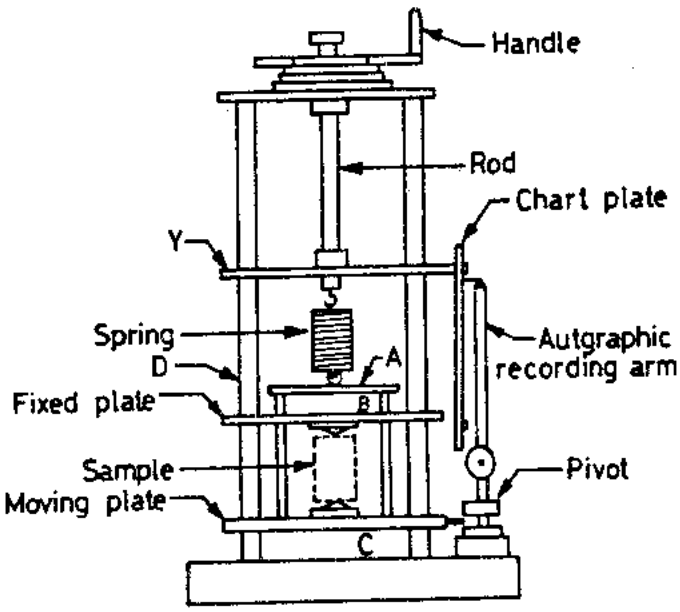


Fig. 13.27. Unconfined Compression Testing Machine (Spring Type).

(2) Machine with a Proving Ring. In this type of the unconfined compression testing machine, a proving ring is used to measure the compressive force (Fig. 13.28). There are two plates, having cone seatings for the specimen. The specimen is placed on the bottom plate so that it makes contact with the upper plate. The dial gauge and proving ring are set to zero.

The compressive load is applied to the specimen by turning the handle. As the handle is turned, the upper plate moves downward and causes compression. (In some machines, the upper plate is fixed and the compressive load is applied by raising the lower plate). The handle is turned gradually so as to produce an axial strain of 1/2% to 2% per minute. The shearing is continued till the specimen fails or till 20% of the axial strain occurs, whichever is earlier.

The compressive force is determined from the proving ring reading, and the axial strain is found from the dial gauge reading.

Presentation of Results. In an unconfined compression test, the minor principal stress (σ_3) is zero. The major principal stress (σ_1) is equal to the deviator stress, and is found from Eq. 13.21.

$$\sigma_1 = P/A$$

where P = axial load,

and A = area of cross-section.

The axial stress at which the specimen fails is known as the unconfined compressive strength (q_u). The stress- strain curve can be plotted between the axial stress and the axial strain at different stages before failure.

While calculating the axial stress, the area of cross-section of the specimen at that axial strain should be used. The corrected area can be obtained from Eq. 13.20 as

$$A = A_0/(1-\varepsilon)$$

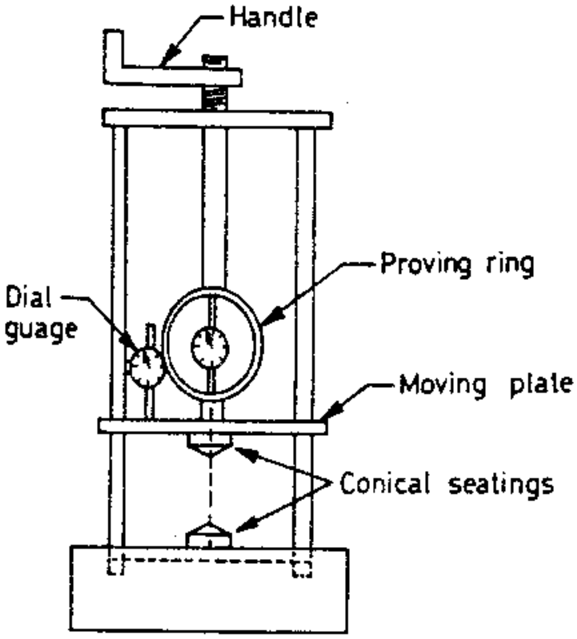


Fig. 13.28. Unconfined Compression Testing Machine (Proving Ring Type).

The Mohr circle can be drawn for stress conditions at failure. As the minor principal stress is zero, the Mohr circle passes through the origin (Fig. 13.29). The failure envelope is horizontal ($\phi_u = 0$). The cohesion intercept is equal to the radius of the circle, *i.e.*

$$s = c_u = \frac{\sigma_1}{2} = \frac{q_u}{2} \qquad \ldots(13.25)$$

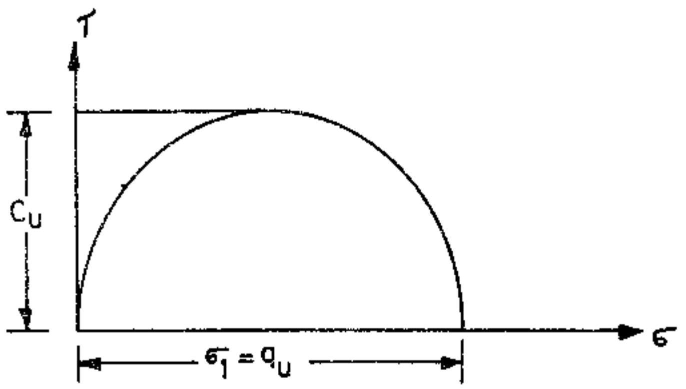


Fig. 13.29. Mohr Circle for Unconfined Compression Test.

Merits and Demerits of the test

Merits

(1) The test is convenient, simple and quick.

(2) It is ideally suited for measuring the unconsolidated-undrained shear strength of intact, saturated clays.

(3) The sensitivity of the soil may be easily determined by conducting the test on an undisturbed sample and then on the remoulded sample.

Demerits

(1) The test cannot be conducted on fissured clays.

(2) The test may be misleading for soils for which the angle of shearing resistance is not zero. For such soils, the shear strength is not equal to half the compressive strength.

(See Chapter 30, Sect. 30.17 for the laboratory experiment).

13.23. VANE SHEAR TEST

The undrained shear strength of soft clays can be determined in a laboratory by a vane shear test. The test can also be conducted in the field on the soil at the bottom of a bore hole. The field test can be performed even without drilling a bore hole by direct penetration of the vane from the ground surface if it is provided with a strong shoe to protect it.

The apparatus consists of a vertical steel rod having four thin stainless steel blades (vanes) fixed at its bottom end. IS : 2720—XXX—1980 recommends that the height H of the vane should be equal to twice the overall diameter D. The diameter and the length of the rod are recommended as 2.5 mm and 60 mm respectively. Fig. 13.30 (a) shows a vane shear test apparatus.

For conducting the test in the laboratory, a specimen of the size 38 mm diameter and 75 mm height is taken in a container which is fixed securely to the base. The vane is gradually lowered into the specimen till the top of the vane is at a depth of 10 to 20 mm below the top of the specimen. The readings of the strain indicator and torque indicator are taken.

Torque is applied gradually to the upper end of the rod at the rate of about 6° per minute (*i.e.* 0.1° per second). The torque acting on the specimen is indicated by a pointer fixed to the spring. The torque is continued till the soil fails in shear. The shear strength of the soil is determined using the formula derived below.

Derivation of Formula. In the deviation of the formula, it is assumed that the shear strength (s) of the soil is constant on the cylindrical sheared surface and at the top and bottom faces of the sheared cylinder. The torque applied (T) must be equal to the sum of the resisting torque at the sides (T_1) and that at the top and bottom (T_2). Thus,

$$T = T_1 + T_2 \qquad \ldots(a)$$

The resisting torque on the sides is equal to the resisting force developed on the cylindrical surface multiplied by the radial distance. Thus,

$$T_1 = (s\pi DH) \times D/2 \qquad \ldots(b)$$

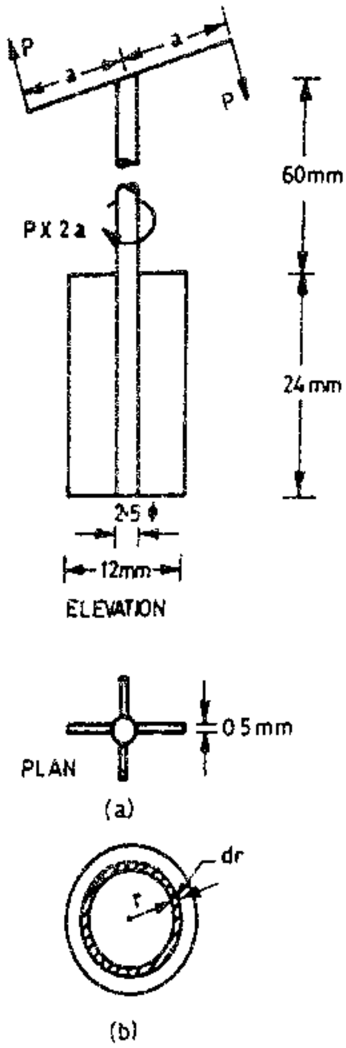


Fig. 13.30. Vane Shear Test.

The resisting torque T_2 due to the resisting forces at the top and bottom of the sheared cylinder can be determined by the integration of the torque developed on a circular ring of radius r and width dr [Fig. 13.30 (b)]. Thus.

$$T_2 = 2\int_0^{D/2} [s\,(2\pi r)\,dr]\,r = 4\pi s\left[\frac{r^3}{3}\right]_0^{D/2}$$

or

$$T_2 = \pi s\,\frac{D^3}{6} \qquad \ldots(c)$$

From Eqs. (a), (b) and (c), $T = \pi s\,[D^2H/2 + D^3/6]$

or $$s = \frac{T}{\pi (D^2 H/2 + D^3/6)} \qquad \text{...(13.27)}$$

For example, if D = 1.2 cm, and H = 2.4 cm, $s = 0.158\ T$

where T is in N-cm and s in N/cm^2.

Eq. 13.27 is modified if the top of the vane is above the soil surface and the depth of the vane inside the sample is H_1. In such a case.

$$s = \frac{T}{\pi (D^2 H_1/2 + D^3/12)} \qquad \text{...(13.28)}$$

The shear strength of the soil under undrained conditions is equal to the apparent cohesion c_u.

The vane shear test can be used to determine the sensitivity of the soil. After the initial test, the vane is rotated rapidly through several revolutions such that the soil becomes remoulded. The test is repeated on the remoulded soils and the shear strength in remoulded state is determined. Thus,

$$\text{Sensitivity } (S_t) = \frac{(s) \text{ undisturbed}}{(s) \text{ remoulded}}$$

Merits and Demerits of Shear Vane Test

Merits.

(1) The test is simple and quick.

(2) It is ideally suited for the determination of the in-situ undrained shear strength of non-fissured, fully saturated clay.

(3) The test can be conveniently used to determine the sensitivity of the soil.

Demerits.

(1) The test cannot be conducted on the fissured clay or the clay containing sand or silt laminations.

(2) The test does not give accurate results when the failure envelope is not horizontal.

13.24. PORE PRESSURE PARAMETERS

A knowledge of the pore water pressure is essential for the determination of effective stresses from the total stresses. The pore water pressure is usually measured in the field by installing piezometers. However, in some cases, it becomes difficult and impractical to install the piezometers and measure the pore water pressure directly in the field. For such cases, a theoretical method for the determination of the pore water pressure is useful. Skempton gave the pore pressure parameters which express the response of pore pressure due to changes in the total stresses under undrained conditions. These parameters are used to predict pore water pressure in the field under similar conditions. The expressions for pore pressure parameters are derived separately for isotropic consolidation, for deviatoric stress and for the combined effect.

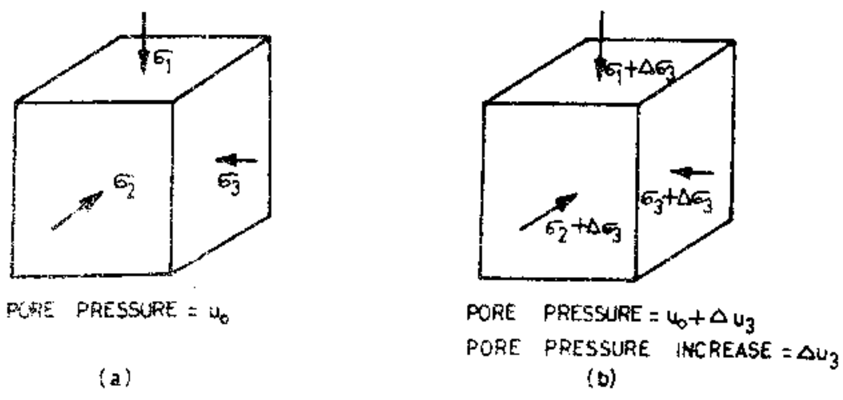


Fig. 13.31. Pore Pressure Under Isotropic Consolidation.

where K is known as Hvorslev coefficient of cohesion. Accordingly, the shear strength can be expressed as

$$s = K\bar{\sigma}_e + \bar{\sigma} \tan \varphi_e \qquad ...(13.52)$$

Bishop and Henkel (1962) suggested a method for determination of c_e and ϕ_e from a series of consolidated-undrained triaxial tests on normally consolidated and over-consolidated specimens. The two failure envelopes are obtained as usual and are shown in Fig. 13.42 (*b*). The water content at failure for the two types of specimens is plotted against the maximum principal stress as shown in Fig. 13.42 (*a*).

For determination of the true failure envelope, any circle (say left circle I) for the over-consolidated clay in Fig. 13.42 (*b*) is chosen. The point corresponding to its maximum stress $(\bar{\sigma}_1)_I$ is projected upward to the $\bar{\sigma}_1 - w_f$ curve in Fig. 13.42 (*a*) to get the point 1 on the curve for over-consolidated clay. The point 1 is projected horizontally across at constant water content to obtain point 2 on the curve for the normally consolidated clay. The point 2 is projected downward to obtain the point $(\bar{\sigma}_1)_{II}$ in Fig. 13.42 (*c*). Through this point, a Mohr circle II is drawn to touch the failure envelope for normally consolidated clay. In Fig. 13.42 (*c*), the left circle I is the same as the circle I in Fig. 13.42 (*b*). The common tangent to the circle I and II in Fig. 13.42 (*c*) is the true failure envelope. The parameters c_e and ϕ_e are obtained from this envelope.

The true failure envelope has been obtained using the concept that two samples can exist at the same water content, one as normally consolidated and one as over-consolidated. As the water contents at points 1 and 2 are equal, the true cohesion is the same and the difference between the shear strength of the two samples is due to the internal friction only.

The fundamental properties of soils can be studied in terms of Hvorslev shear strength parameter. However, the theory is generally used only for research purposes. For practical use in engineering problems, the Mohr-Coulomb theory is commonly used.

13.30. LIQUEFACTION OF SANDS

As discussed earlier, the shear strength of sandy soils is given by the Mohr-Coulomb equation (Eq. 13.13), taking the cohesion intercept as zero.

Thus $$s = \bar{\sigma} \tan \phi' \qquad ....(13.53)$$

If the sand deposit is at a depth of z below the ground and the water table is at the ground surface, the effective stress is given by (see Chapter 10),

$$\bar{\sigma} = \gamma_{sat}\, z - \gamma_w z = \gamma' z$$

Therefore, $$s = \gamma' z \tan \phi'$$

If the sand deposit is shaken due to an earth-quake or any other oscillatory load, extra pore water pressure (u') develops, and the strength equation becomes

$$s = (\gamma' z - u') \tan \phi'$$

It can also be expressed in the term of extra pore pressure head h, where $u' = \gamma_w h$. Thus

$$s = (\gamma' z - \gamma_w h) \tan \phi' \qquad ...(13.54)$$

As indicated by Eq. 13.54, the shear strength of sand decreases as the pore water increases. Ultimately, a stage is reached when the soil loses all its strength. In which case,

$$\gamma' z - \gamma_w h = 0$$

or $$\frac{h}{z} = \frac{\gamma'}{\gamma_w}$$

Expressing h/z as critical gradient,

$$i_{cr} = \frac{(G-1)\,\gamma_w}{1+e} \cdot \frac{1}{\gamma_w}$$

or $$i_{cr} = \frac{G-1}{1+e} \qquad ...(13.55)$$

The phenomenon when the sand loses its shear strength due to oscillatory motion is known as *liquefaction of sand*. The structures resting on such soils sink. In the case of partial liquefaction, the structure may undergo excessive settlement and the complete failure may not occur.

The soils most susceptible to liquefaction are the saturated, fine and medium sands of uniform particle size. When such deposits have a void ratio greater than the critical void ratio and are subjected to a sudden shearing stresses, these decrease in volume and the pore pressure u' increases. The soil momentarily liquefies and behaves as a dense fluid. Extreme care shall be taken while constructing structures on such soils. If the deposits are compacted to a void ratio smaller than the critical void ratio, the chances of liquefaction are reduced. (See Chapter 32 for more details on liquefaction of sand.)

13.31. SHEAR CHARACTERISTICS OF COHESIONLESS SOILS

The shear characteristics of cohesionless soils can be summarized as given below.

The shear strength of cohesionless soils, such as sands and non- plastic silts, is mainly due to friction between particles. In dense sands, interlocking between particles also contributes significantly to the strength.

The stress-strain curve for dense sands exhibits a relatively high initial tangent modulus. The stress reaches a maximum value at its peak at a comparatively low strain and then decreases rapidly with an increasing strain and eventually becomes more or less constant, as discussed earlier. The stress-strain curve for loose sands exhibits a relatively low initial tangent modulus. At large strains, the stress becomes more or less constant.

The dense sand shows initially a volume decrease in a drained test, but as the strain increases, the volume starts increasing. The loose sand shows a volume decrease throughout.

In the case of loose sand, the specimen bulges and ultimately fails by sliding simultaneously on numerous planes. The failure is known as the *plastic failure* [Fig. 13.43 (*a*)]. In the case of dense sand, the specimen shows a clear failure plane and the failure is known as the *brittle failure* [Fig. 13.43 (*b*)].

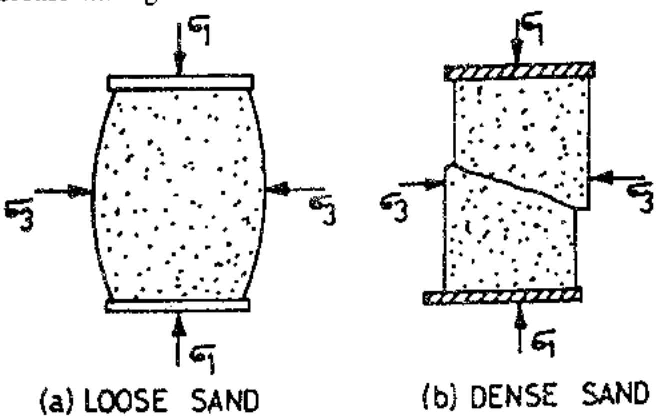


Fig. 13.43. Types of Failure.

The failure envelope for dense sand can be drawn either for the peak stresses or for the ultimate stresses. The value of the angle of shearing resistance (ϕ') for the failure envelope for peak stresses is considerably greater than that for the ultimate stresses. In the case of loose sands, as the peak stress and the ultimate stress are identical, there is only one failure envelope. The angle of shearing resistance in very loose state is approximately equal to the angle of repose. The angle of repose is the angle at which a heap of dry sand stands without any support. It has been established that air-dry sand gives approximately the same value of ϕ' as the saturated sand. As it is easier to perform tests on dry sand, tests can be performed on dry sand instead of saturated sand.

If the failure envelope is slightly non-linear, a straight line may be drawn for the given pressure range and the angle of shearing resistance is taken as the slope of this line. The cohesion intercept, if any, is usually neglected.

The angle of shearing resistance of sands in the field can be determined indirectly by conducting in-situ tests, such as the standard penetration test (SPT) as explained in chapter 17.

The factors that affect the shear strength of cohesionless soils are summarized below:

(1) **Shape of particles.** The shearing strength of sands with angular particles having sharp edges is greater than that with rounded particles, other parameters being identical.

(2) **Gradation.** A well-graded sand exhibits greater shear strength than a uniform sand.

(3) **Denseness.** The degree of interlocking increases with an increase in density. Consequently, the greater the denseness, the greater the strength. The value of ϕ' is related to the relative density (D_r) as $\phi' = 26° + 0.2\,D_r$. However, the ultimate value of ϕ' is not affected by denseness.

(4) Confining pressure. The shear strength increases with an increase in confining pressure. However, for the range of pressures in the common field problems, the effect of confining pressure on the angle of shearing resistance is not significant.

(5) Deviator stress. The angle ϕ' decreases under very high stresses. As the maximum deviator stress is increased from 500 to 5000 kN/m^2, the value of ϕ' decreases by about 10%. This is due to the crushing of particles.

(6) Intermediate principal stress. The intermediate principal stress affects the shear strength to a small extent. The friction angle for dense sands in the plane strain case is about 2° to 4° greater than that obtained from a standard triaxial test. However, for loose sand, there is practically no difference in the two values.

(7) Loading. The angle of shearing resistance of sand is independent of the rate of loading. The increase in the value of ϕ' from the slowest to the fastest possible rate of loading is only about 1 to 2%.

The angle of shearing resistance in loading is approximately equal to that in unloading.

(8) Vibrations and Repeated loading. Repeated loading can cause significant changes. A stress much smaller than the static failure stress if repeated a large number of times can cause a very large strain and hence the failure.

(9) Type of minerals. If the sand contains mica, it will have a large void ratio and a lower value of ϕ'. However, it makes no difference whether the sand is composed of quartz or feldspar minerals.

(10) Capillary moisture. The sand may have apparent cohesion due to capillary moisture. The apparent cohesion is destroyed as soon as the sand becomes saturated.

A person can easily walk on damp sand near the sea beach because it possesses strength due to capillary moisture. On the same sand in saturated conditions, it becomes difficult to walk as the capillary action is destroyed.

Table 13.2 gives the representative values of ϕ' for different types of cohesionless soils.

Table 13.2. Representative Values of ϕ' for Sands and Silts

S. No.	Soil	ϕ'
1.	Sand, round grains, uniform	27° to 34°
2.	Sand, angular, well-graded	33° to 45°
3.	Sandy gravels	35° to 50°
4.	Silty sand	27° to 34°
5.	Inorganic silt	27° to 35°

Note. Smaller values are for loose conditions and larger values are for dense conditions.

13.32. SHEAR CHARACTERISTICS OF COHESIVE SOILS

The shear characteristics of cohesive soils are summarized below :

The shear characteristics of a cohesive soil depend upon whether a soil is normally consolidated or over-consolidated. The stress- strain curve of an over-consolidated clay is similar to that of a dense sand and that of a normally consolidated clay is identical to that of a loose sand. However, the strain required to reach peak stress are generally greater in clay than in sand. The high strength at the peak point in an over-consolidated clay is due to structural strength; whereas in the dense sand, it is mainly due to interlocking. In over-consolidated clay, strong structural bonds develop between the particles. Loose sands tend to increase in volume at large strains whereas normally consolidated clays show no tendency to expand after a decrease in volume.

The effective stress parameters (c', ϕ') for an overconsolidated clay are determined from the failure envelope,

$$s = c' + \bar{\sigma} \tan \phi'$$

However, for a normally consolidated clay, the failure envelope passes through the origin and hence $c' = 0$.